J. Phys. B: At. Mol. Opt. Phys. **46** (2013) 185401 (5pp) doi:10.1088/0953-4075/46/18/185401

Inverse Smith–Purcell effect near rough surfaces

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Received 7 March 2013, in final form 9 July 2013
Published 5 September 2013
Online at stacks.iop.org/JPhysB/46/185401

Abstract
Absorption of a photon by an electron moving parallel to a rough surface is studied. In the weak scattering regime we have evaluated the absorption probability of absorption of a single photon of energy ω. It is shown that the absorption probability with diffusional contribution becomes large by a $l_{\text{in}}/l \gg 1$ factor compared to the analogous result with the single scattering contribution. The maximum of probability takes place for the infrared wavelengths and strongly depends on the particle energy. We also discuss the case of two-dimensional periodical surface profile and indicate optimal conditions for maximal absorption probability. The results can be used in electron energy gain spectroscopy and in laser-driven acceleration.

1. Introduction

It is well known that a charged particle moving in the vacuum cannot emit or absorb photons due to the energy–momentum conservation laws. On the other hand, emission or absorption is possible when a particle moves in a medium or close to an interface. Cherenkov, transition and Smith–Purcell radiations are examples of the above mentioned emission (see, for example, [1]). In recent years the inverse counterparts of the mentioned radiations have been observed [2–5]. Earlier the inverse Smith–Purcell effect for periodical metallic gratings was theoretically analysed in the sub-millimeter wavelength region [6]. Interest in these effects is largely motivated by a possibility of the laser-driven acceleration of charged particles. Besides, the inverse Smith–Purcell effect can be used in electron energy gain spectroscopy [7].

In the present paper we investigate the inverse Smith–Purcell effect, namely the absorption of a photon by an electron which moves parallel to a rough surface. To the best of our knowledge, no such calculations have been previously reported. The main difficulties with rough surfaces arise because it is more difficult to perform analytical calculations when we deal with an arbitrary shaped profile of the grating for understandable reasons: there is no general algorithm to calculate the radiating part in the reflected waves. At present, most numerical simulations are available as one of the effective tools to analyse and to observe a variety of physical quantities such as electromagnetic fields as functions of time and space, power outflow, radiated intensity as a function of the radiating angle, etc (see, e.g., [8] and references therein). Therefore any study of the inverse Smith–Purcell radiation from rough surfaces should be quite important and analytical results are highly desirable. The purpose of the present work goes in this direction, in the sense that we provide an analytical expression for the absorption probability of a photon by an electron moving parallel to a rough surface. In the diffusion regime we were able to obtain a closed analytic expression for the absorption probability, taking into account the diffusion contribution. We show that the diffusion contribution is dominant compared to the single scattering probability.

The radiation from a charged particle moving parallel to rough surfaces has been considered recently by one of the authors (Gevorkian) [10, 11]. The averaged radiation intensity for a quite general surface random profile was directly calculated and it is shown that the main contribution to the radiation intensity is determined by the multiple scattering of polaritons induced by a charge on the surface. We will develop an approach, following closely [10, 11], which allows us to investigate periodical as well as random surface profiles, different materials from sub-millimeters to optics. We indicate the necessary conditions for absorption to take place.

This work is organized as follows. In section 2 we briefly formulate the problem and introduce the basic equation for the absorption probability of a single photon of energy ω. In section 3 we carry out an exhaustive description of our

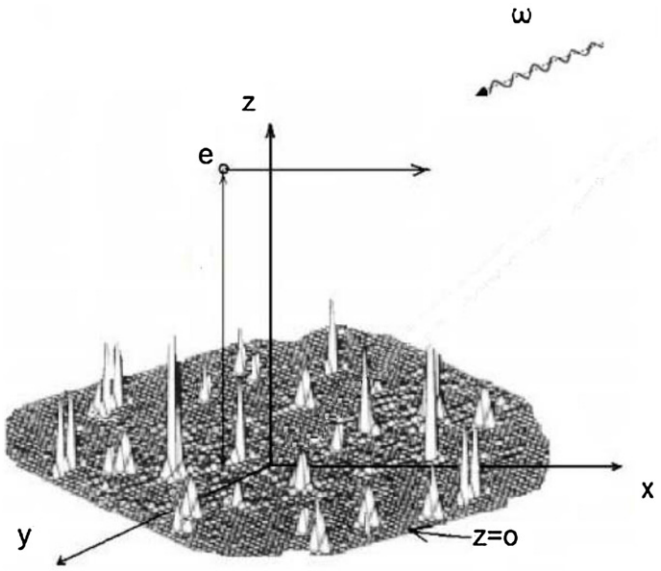

Figure 1. Geometry of the problem. The electron moves parallel to the rough surface at the plane xy which is illuminated by an external laser field.

two-dimensional rough surface and the analytical approach used in absorption probability calculations. In section 4 we calculate the absorption probabilities with single and multiple scattering contributions. It is shown that the diffusion contribution to the absorption probability is the dominant one. In section 5 we discuss the utilization of the inverse Smith–Purcell effect for particle acceleration. Finally, we summarize our results in section 4.

2. Initial relations

Suppose that a fast electron moves on the positive x direction parallel to a rough surface xy at the distance Z from it. Simultaneously a laser field of frequency ω falls down on the surface, see figure 1.

The electron wave function can be described as

$$\Phi(\vec{r}) = \frac{1}{\sqrt{L_x}}\varphi(Z,Y)\,\mathrm{e}^{\frac{\mathrm{i}p_i x}{\hbar}}, \tag{1}$$

where L_x is the system size in the $0x$ direction, $\varphi(Z,Y)$ is the wave function in the zy plane, p_i is the electron momentum along the direction of motion. After absorbing a photon electron momentum and energy become $p_f = p_i + \hbar q$ and $E_f = E_i + \hbar q v$, respectively (v is the velocity of the electron and a non-recoil approximation $\hbar q \ll p_i$ is assumed). For a fast electron one can assume that the wave function $\varphi(Z,Y)$ remains unchanged during the interaction with the photon. We will discuss the applicability conditions of this assumption below. Treating the electron and photon as quantum mechanical subjects, the absorption probability of a single photon of energy ω can be represented by the form [7]

$$P(\omega) = \left(\frac{e}{\hbar\omega}\right)^2 \left|\int \mathrm{d}x\, E_x(x,Y,Z)\,\mathrm{e}^{-\mathrm{i}\frac{\omega x}{v}}\right|^2. \tag{2}$$

$E_x(x,Y,Z)$ is the electric field component along the electron motion direction that includes incident to as well as scattered from the surface fields and Y, Z are the electron constant coordinates in the perpendicular to motion plane. If the incident field is a plane wave then it is easy to convince oneself that the incident part does not contribute to the integral. Therefore, for analytical evaluation of the integral, equation (2), the field E_x will be substituted by the scattered one. This will be carried out in the next sections, separately for the situations when single and multiple scattering contributions are taken into account, while calculating the appropriate absorption probability.

3. Scattered field

Dielectric constant of the system is described as $\varepsilon(\vec{r}) = \theta(z - h(x,y)) + \varepsilon(\omega)\theta(h(x,y) - z)$, where θ is the step function and $\varepsilon(\omega)$ is the dielectric constant of the isotropic medium, $h(x,y)$ is the random profile of the surface. Assuming that h is small and expanding the $\varepsilon(\vec{r})$ in powers of h and keeping linear terms we get $\varepsilon(\vec{r}) = \varepsilon_0(z) + \varepsilon_r(\vec{r})$, where $\varepsilon_r(\vec{r}) = (\varepsilon - 1)h(x,y)\delta(z)$. The function $\varepsilon_0(z) = 1$ at $z > 0$ and $\varepsilon_0(z) = \varepsilon(\omega)$ at $z < 0$ describes the flat surface between vacuum and medium. Under this assumption, the scattered electric field can be represented as follows:

$$E_{\mu s}(\vec{r}) = -\frac{\omega^2}{c^2}(\varepsilon - 1)\int \mathrm{d}\vec{r}' G_{\mu\nu}(\vec{r},\vec{r}',\omega)h(\vec{\rho}')\delta(z')E^0_\nu(\vec{r}',\omega). \tag{3}$$

$E^0_\nu(\vec{r}',\omega)$ is the solution of the Maxwell equation with the flat interface and because of the translational symmetry in the xy plane it can be represented as follows: $E^0_\nu(\vec{r}) = e^{\vec{k}_{||}\vec{\rho}}E^0_\nu(z)$, where $\vec{k}_{||}$ and $\vec{\rho}$ are two-dimensional vectors in the xy plane. Green's function in equation (3) obeys the inhomogeneous Maxwell equation:

$$\left[\varepsilon_0(z)\frac{\omega^2}{c^2}\delta_{\lambda\mu} - \frac{\partial^2}{\partial r_\lambda \partial r_\mu} + \delta_{\lambda\mu}\nabla^2 + \varepsilon_r(\vec{r})\frac{\omega^2}{c^2}\delta_{\lambda\mu}\right] \times G_{\mu\nu}(\vec{r},\vec{r}',\omega) = \delta_{\lambda\nu}\delta(\vec{r} - \vec{r}'). \tag{4}$$

It is worth noticing that the presence of the δ-function in the expression of ε_r will lead to different values of any physical quantity at $z = 0$, while evaluating the integral over z. To avoid the problem with discontinuous physical quantities at $z = 0$ in our further calculations we will take their value at $z = 0^+$, see also, [12]. Such determination of integrals over δ-functions give correct answers in the limit $|\varepsilon| \to \infty$. Hence, substituting equations (4) and (3) into equation (2), one has

$$P(\omega) = \left(\frac{e}{\hbar\omega}\right)^2 (\varepsilon - 1)^2\frac{\omega^4}{c^4}\int \mathrm{d}x\,\mathrm{d}x'\,\mathrm{d}\vec{\rho}_1\,\mathrm{d}\vec{\rho}_2 \; G_{x\nu}(x,Y,Z,\vec{\rho}_1,0^+)G^*_{\mu x}(\vec{\rho}_2,0^+,x',Y,Z) \times h(\vec{\rho}_1)h(\vec{\rho}_2)E^0_\nu(\vec{\rho}_1,0^+)E^{0*}_\mu(\vec{\rho}_2,0^+)\,\mathrm{e}^{-\mathrm{i}\frac{\omega}{v}(x-x')}. \tag{5}$$

This is a general expression, independent of the model considered and can be applied for both, periodical and random grating cases. Below, we will analyse these cases separately (hereafter the sign + is omitted). In the periodical grating case (photonic crystal), the surface profile is a periodical function $h(\vec{\rho}) = \delta\cos\vec{K}\vec{\rho}$, where $\vec{K} = (2\pi/b, 2\pi/d)$ is a two-dimensional vector and b, d are the grating periods in the x and y directions, respectively. In the rough surface case, $h(\vec{\rho})$ is a Gaussian distributed random function. First let us consider the

photonic crystal case. It is convenient first to present Green's function in the form:

$$G_{\mu\nu}(\vec{r},\vec{r}') = \int G_{\mu\nu}(\vec{p}|z,z')\,\mathrm{e}^{\mathrm{i}\vec{p}(\vec{\rho}-\vec{\rho}')}\frac{\mathrm{d}\vec{p}}{(2\pi)^2}, \tag{6}$$

where $G_{\mu\nu}(\vec{p}|z,z')$ is the Fourier transform in the xy plane. In the second step, let us assume that the plane of incidence of external light is xz. Then the background electric field in equation (5) that includes incident and reflected parts, takes the form: $E_\nu(\vec{\rho},0+) = \mathrm{e}^{\mathrm{i}k_x x}E_\nu$, where $k_x = \omega\cos\theta/c$ and θ is the angle between the external photon momentum and electron velocity directions. Substituting expressions for Green's function, electric field and $h(\vec{\rho})$ into equation (5), one finds

$$P(\omega) = \frac{g_1\pi L_x}{2}G_{x\nu}\left(\frac{\omega}{v},\frac{2\pi}{d}|Z,0\right)G^*_{\mu x}\left(-\frac{\omega}{v},-\frac{2\pi}{d}|0,Z\right) \times \delta\left(k_x+\frac{2\pi}{b}-\frac{\omega}{v}\right)E_\nu E^*_\mu, \tag{7}$$

where $g_1 = (e/\hbar\omega)^2\frac{\omega^4}{c^4}(\varepsilon-1)^2\delta^2$, L_x is the system size in the x direction and $\delta(k_x=0)$ was substituted by $L_x/2\pi$. In the weak scattering regime $(\varepsilon-1)^2\delta^2/\lambda^2 \ll 1$, Green's functions in equation (7) can be substituted by the bare Green's functions. The latter quantities are the solutions of equation (4) with $\varepsilon_r \equiv 0$ and were found in [12]. The existence of the δ-function in equation (7) sets the relation between the external light wavelength, incident angle, electron velocity and the grating period. Interestingly, they are related to each other in the same way as in the direct Smith–Purcell effect

$$\lambda = b\left(\frac{1}{\beta}-\cos\theta\right), \tag{8}$$

with $\beta = v/c$. Note that the dispersion relation depends only on the grating period in the electron velocity direction. As shown in [12] $G_{xy} = G_{yx} \equiv 0$ which for our problem means that a photon polarized in the perpendicular to incidence plane (s-polarization) cannot be absorbed by the electron. Therefore we will consider the case when the incident photon is p-polarized. To simplify the problem consider the limit $|\varepsilon| \gg 1$. In this case the main contribution to equation (7) comes from the term containing G_{xz}. The explicit expression of the bare Green's function is (see [12])

$$G_{xz}(\vec{p}|z,0) = -G_{zx}(\vec{p}|0,z) = -\frac{\mathrm{i}p_x}{k^2}\frac{\varepsilon(\omega)q\,\mathrm{e}^{\mathrm{i}qz}}{k_1-\varepsilon(\omega)q}, \tag{9}$$

where $q = \sqrt{k^2-p^2}$ if $k^2 > p^2$ and $i\sqrt{p^2-k^2}$ if $k^2 < p^2$ and $k_1 = -(\varepsilon(\omega)k^2-p^2)^{1/2}$. Substituting equation (9) into equation (7), one comes after some algebraic manipulations to the following expression for $P(\omega)$

$$P(\omega) = \frac{\pi g_1 L_x}{2} \times \frac{c^2|\varepsilon(\omega)|^2\left(\gamma^{-2}+\frac{\lambda^2\beta^2}{d^2}\right)|E_z|^2\delta\left(\frac{\omega}{v}-\frac{2\pi}{b}-k_x\right)}{\beta^2\omega^2\left[\left(\varepsilon(\omega)\beta^2-1-\frac{\lambda^2\beta^2}{d^2}\right)^{1/2}+\mathrm{i}\varepsilon(\omega)\sqrt{\gamma^{-2}+\frac{\lambda^2\beta^2}{d^2}}\right]} \times \frac{\exp\left(-4\pi Z\sqrt{\frac{1}{d^2}+\frac{1}{\gamma^2\beta^2\lambda^2}}\right)}{\left[\left(\varepsilon^*(\omega)\beta^2-1-\frac{\lambda^2\beta^2}{d^2}\right)^{1/2}-\mathrm{i}\varepsilon^*(\omega)\sqrt{\gamma^{-2}+\frac{\lambda^2\beta^2}{d^2}}\right]}, \tag{10}$$

with $\gamma = (1-\beta^2)^{-1/2}$. The amplitude of electric field includes both incident and reflected parts, $E_z = (1+r(\omega))E^1_z$, where E^1_z is the amplitude of incident field and $r(\omega)$ is the reflection amplitude that goes to unity in the limit $|\varepsilon| \to \infty$. Because of the exponential function absorption takes place for the electron distances from the surface satisfying the condition:

$$4\pi\left(\frac{1}{d^2}+\frac{1}{\lambda^2\beta^2\gamma^2}\right)^{1/2}Z \ll 1. \tag{11}$$

For the distances, defined by equation (11), the electron transverse wave function $\varphi(Z,Y)$ remains unchanged during the interaction with the photon. We need this assumption for derivation of absorption probability equation (2). The essential absorption probability is achieved in the case when the imaginary part of $\varepsilon(\omega)$ is small compared to the real part. Such a situation occurs, for example, for noble metals Au, Ag, Cu, etc at the infrared wavelengths [9]. As an example, we note that for gold at photon energy $\hbar\omega = 1$ eV, $\mathrm{Re}\,\varepsilon = -70$ and $\mathrm{Im}\,\varepsilon = 6.27$. Assuming $\varepsilon(\omega)$ is real, for the distances, equation (11), one obtains

$$P(\omega) = \frac{\pi g_1 L_x}{2} \times \frac{c^2\varepsilon^2(\omega)\left(\gamma^{-2}+\frac{\lambda^2\beta^2}{d^2}\right)|E_z|^2\delta\left(\frac{\omega}{v}-\frac{2\pi}{b}-k_x\right)}{\beta^2\omega^2\left[\varepsilon(\omega)\beta^2-1-\frac{\lambda^2\beta^2}{d^2}+\varepsilon^2(\omega)\left(\gamma^{-2}+\frac{\lambda^2\beta^2}{d^2}\right)\right]}. \tag{12}$$

Since the absorption probability is positive one gets a condition on the particle velocity

$$\frac{v^2}{c^2} \leqslant \frac{(\varepsilon+1)}{\varepsilon-\frac{\lambda^2(\varepsilon+1)}{d^2}}. \tag{13}$$

The maximum of $P(\omega)$ is achieved provided that the equality in equation (13) holds. It follows from equation (13) that the wavelength $\lambda \gg d$ is suppressed. Taking the limit $d \to \infty$ one returns, as it should be expected, to the Smith–Purcell geometry with periodicity only in the direction of particle motion. In this geometry and for metallic gratings ($\varepsilon < -1$), the quantity $c\sqrt{(\varepsilon+1)/\varepsilon}$ represents the velocity of induced polariton on the surface. Hence, the condition, equation (13), means that the maximum of absorption is reached if the particle velocity equals the polariton velocity.

4. Rough surface

The above consideration can be generalized to a case when a charged particle travels over a rough surface, by applying a recently developed approach to study the spectrum of radiation from a surface roughness [10, 11]. It can be shown that in this case absorption probability consists of two parts, each of them has a different origin and must be evaluated separately. One is caused by the single scattering of polaritons, another is caused by their diffusion, i.e., by the multiple scattering effect. Using equation (5), for the single scattering contribution to the absorption probability, one obtains

$$P^r(\omega) = g_1\int \mathrm{d}x\,\mathrm{d}x'\,\mathrm{d}\vec{\rho}_1\vec{\rho}_2 G_{x\nu}(x,Y,Z,\vec{\rho}_1,0^+) \times G^*_{\mu x}(\vec{\rho}_2,0^+,x',Y,Z) \times W(|\vec{\rho}_1-\vec{\rho}_2|)E_\nu(\vec{\rho}_1)E^*_\mu(\vec{\rho}_2)\,\mathrm{e}^{-\mathrm{i}\frac{\omega(x-x')}{v}} \tag{14}$$

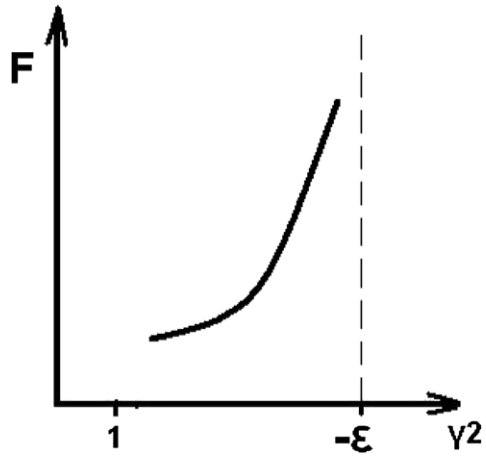

Figure 2. Absorption probability dependence on the particle energy.

where $\delta^2 W(|\vec{\rho}_1 - \vec{\rho}_2|) = \langle h(\vec{\rho}_1)h(\vec{\rho}_2)\rangle$ is the correlation function of the rough surface profile. We assume that the surface profile fluctuations are uncorrelated. This allows us to substitute the correlation function W by the δ-function except the cases when finite correlation length is needed for divergence reasons, i.e. to avoid the divergences in integral calculations. Such an approximation is justified provided that $\lambda \gg \sigma$, where σ is the correlation length of random surface profile fluctuations. Next, we evaluate the integral equation (15), by assuming $W(p) = \pi\sigma^2 e^{-p^2\sigma^2/4}$. Then, taking the Fourier transforms, substituting equation (9) into equation (15) in the limit $Z \to 0$ (more precisely $Z \ll \beta\lambda\gamma/2\pi$), we obtain the desired result for the absorption probability with single scattering contribution

$$P^r(\omega) = \frac{\pi L_x g(1+r)(1+r^*)|E_z|^2 c}{\beta^4\omega} F(\varepsilon,\beta) \tag{15}$$

where

$$F(\varepsilon,\beta) = \frac{\varepsilon^2}{(\varepsilon^2-1)}\left[1 + \frac{\omega\pi\sigma\varepsilon\beta}{2c\sqrt{(1+\varepsilon)(1+\varepsilon\gamma^{-2})}}\right]. \tag{16}$$

Note that in equations (15), (16), like before, we assume that the imaginary part of ε is small and neglect it compared to the real part. For the positive ε (dielectrics), as follows from $F(\varepsilon,\beta)$, the expression under the square root is always positive. As for negative ε (metals), it can be easily checked, whether it leads to serious restriction on the energy of particle, i.e. $\gamma^2 \leqslant -\varepsilon$. Maximum absorption is achieved when the equality holds, i.e., $\gamma^2 = -\varepsilon(\omega)$, see figure 2. For the negative ε, a plasmon-polariton is formed on the surface, see for example [13]. The pole at $p^2 = \varepsilon k^2/(\varepsilon+1)$ in the Green's function equation (9) is manifestation of the plasmon-polariton. It is scattered on the inhomogeneities and gives contribution to the scattered electric field in equation (2) and hence to the absorption probability $P(\omega)$. The expression (15) is the plasmon–polariton single scattering contribution. To make further analytical progress in the study of $P(\omega)$ we will assume that the following inequality is met: $\lambda \ll l \ll l_{\rm in}, L$, where $l, l_{\rm in}$ are elastic and inelastic mean free paths of the polariton on the surface. In other words, we will assume that the condition of multiple or diffusional scattering of the polariton are realized in the surface. In our calculations of the diffusional contribution to the absorption probability we follow closely [10, 11]. Further manipulations are completely analogous to those outlined in [10, 11] for the case of the radiation problem. Hence, here we present the final result without derivation by noting that the diffusion contribution to the absorption probability is the dominant one

$$P^D(\omega) = \frac{32}{3}\frac{l_{\rm in}}{l}P^r(\omega). \tag{17}$$

Indeed, as seen from equation (17), the quantity $P^D(\omega)$ is proportional to $P^r(\omega)$, with prefactor $l_{\rm in}/l$, which is the average number of polariton scatterings in the system. In the diffusion regime the ratio is a large number, i.e. $l_{\rm in}/l \gg 1$, see also [14], justifying that the diffusion contribution is dominant. It is important to notice that one of the advantages of the random surface profile is that the external light incident angle can be arbitrary instead of a certain one in the periodical case.

For completeness, we also compare the absorption probability with the probability of emission of a photon by a charge particle moving under the same conditions, see figure 1. The probability of emission of a photon of energy ω by an electron moving over a rough surface can be estimated as (following [10, 11])

$$P^e(\omega) \approx \frac{2e^2}{3\hbar c\beta^2} g_0(\omega)\frac{L_x}{Z}\frac{l_{\rm in}(\omega)}{l(\omega)}, \tag{18}$$

where $g_0 = (\varepsilon-1)^2 k^4\delta^2\sigma^2$ and $Z \ll \lambda\beta\gamma/2\pi$ is the distance from the plane $z = 0$. Using equations (15)–(18) the ratio of probabilities can be estimated as

$$R = \frac{P^D(\omega)}{P^e(\omega)} \approx \frac{16Zc^3|E_z|^2(1+r)(1+r^*)}{\hbar\omega^4} F(\varepsilon,\beta). \tag{19}$$

Now let us estimate numerically R. Before doing so, first we verify numerically the applicability of the diffusion approximation. Note, that in the weak scattering regime, average mean free paths are described by the following expressions: $l = 4|\mathrm{Re}\,\varepsilon|/kg_0$ and $l_{\rm in} = (\mathrm{Re}\,\varepsilon)^2/k\,\mathrm{Im}\,\varepsilon$ [10, 11]. For Au at the photon energy $\hbar\omega = 1$ eV $\mathrm{Re}\,\varepsilon = -70$ and $\mathrm{Im}\,\varepsilon = 6.27$, $r = r^* \sim 1$. Taking for the roughness parameters $\delta = 10$ nm and $\sigma = 100$ nm one gets $g_0 \sim 3.13$ and $l \sim 14\lambda$ and $l_{\rm in} \sim 124\lambda$. This means that the conditions $\lambda \ll l \ll l_{\rm in}$ of the diffusion of polaritons are realized in the system. Now taking electron energy $E = 3.5$ Mev, $Z \sim \lambda\beta\gamma/2\pi$, laser power $|E_z|^2 c \sim 10^{10}$ Wm^{-2} one finds from equation (19) that $R \sim 18$.

Note that in contrast to the radiation case [11], where the maximum is achieved for short wavelengths (blue part of the visible region), here the probability maximum takes place for infrared wavelengths. An interesting and different feature of the absorption probability, compared to the radiation case is its strong dependence on the particle energy, figure 2. This dependence can be quantitatively measured and can be used for investigation of metal dielectric constant $\varepsilon(\omega)$ in the optical region.

Concluding this section let us note that the ratio R can be made essentially larger via increasing the laser power. This point is topical for the laser-driven acceleration application of the inverse Smith–Purcell effect.

5. Laser-driven acceleration

We now want to discuss utilization of the inverse Smith–Purcell effect for particle acceleration. Metal surfaces with rough or one-dimensional periodic gratings cannot be used for acceleration purposes because of the restriction on the particle's energy (see equation (13)). However, there is an important exception, when the strength of the electric field, which determines the absorption probability and was scattered from a metal surface, can be resonantly large. To illustrate this, we consider the two-dimensional periodical grating case. The absorption probability, equation (12), is straightforwardly applicable in this case. Rewriting the restriction condition on the energy, equation (13), in the form

$$\gamma^2 \leqslant \frac{\frac{\lambda^2}{d^2}(\varepsilon+1)-\varepsilon}{1+\frac{\lambda^2}{d^2}(\varepsilon+1)}, \tag{20}$$

it is easy to see that the most favourable situation happens when the photon energy satisfies the resonant condition, i.e. the denominator of equation (20) becomes zero

$$1+\left(\frac{2\pi c}{\omega d}\right)^2(\varepsilon(\omega)+1)=0. \tag{21}$$

Note that we have in mind optical frequencies for which $\varepsilon(\omega)$ is a large negative number. In this case the energy of the accelerated particle can be very large. The absorption probability, equation (12), at the resonance photon energy will be large too.

The largeness of the absorption probability is caused by the resonance enhancement of the scattered field due to the surface plasmon–polaritons. The growth of probability is limited only by the losses in the optical region. In dielectrics, the restriction on energy is absent and they can be used for acceleration purposes, see, for example, [15].

6. Summary

In conclusion, we have investigated the absorption of a photon by an electron moving over a rough surface. Optimal conditions that include polarization of incident light, electron energy, material and grating types are indicated. In particular, it is shown that only the p-polarized photon can be absorbed. For metallic surfaces and for relativistic particles two-dimensional periodical grating is preferable because of the restriction on the energy of the particle. For dielectrics, the restriction on the energy is absent.

Acknowledgments

We are grateful to A Akopian and F J de Abajo for helpful comments. VG acknowledges partial support by FEDER and the Spanish DGI under project no. FSI2010-16430.

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