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Localization length in a quasi-one-dimensional disordered system in the presence of an electric field

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Abstract

A two-dimensional δ-potential Kronig–Penney model for quasi-one-dimensional (Q1D) disordered systems is used to study analytically the influence of a constant electric field on the inverse localization length (LL). Based on the Green's function formalism we have calculated LL as a function of the incoming energy E, electric field F, length L of the Q1D sample, number of modes M in the transverse direction and the amount of disorder w. We show that, for large L in Q1D systems, states are weakly localized, i.e. we deal with power-law localization. With increasing electric field in Q1D mesoscopic systems a transition from exponential to a power-law behavior takes place, as in 1D systems. We note that the graphs showing the inverse LL change significantly with increasing F (for fixed M) rather than with increasing M (for fixed F). We also show that the graphs representing the ratio of the corresponding localization length without and with electric field collapse for all modes M into a universal curve in the Q1D strip model.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The problem of localization of the eigenstates in one-dimensional (1D) disordered finite systems in an external electric field F has been studied extensively, both numerically and analytically over the last three decades [1–10]. It has been well established that in 1D systems the electron's transmission coefficient shows a transition from exponential to power-law behavior with increasing electric field. More precisely, it has been shown that the transmission coefficient is a function of a dimensionless parameter $X = FL/E$ (in units of $e = 1$), the ratio of the electrostatic energy FL to the energy E of the incident electron. The parameter X plays an important role in the behavior of electrons and the limiting case of $X \ll 1$ coincides with an exponential localized regime and $X \gg 1$ with a power-law regime. With further increase in X, the system becomes metallic in the sense that all the states are extended or delocalized. The details of these states depend on the parameters F, L, E and w (the measure of disorder) as well as on the form of the scattering potentials (smooth or discontinuous). But the existence of three regimes in 1D disordered systems, i.e. exponentially localized, power-localized and extended states, has already been established [1, 11]. Moreover, in the case of a 1D chain with δ-function potentials, Prigodin [1] and later Soukoulis *et al* [2] have shown that in the approximation of white-noise potential the expression $-\frac{l_0}{2L}\langle \ln T\rangle$ follows a universal form

$$-\frac{l_0}{2L}\langle \ln T\rangle = \frac{\ln(1+X)}{X}, \tag{1}$$

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leading to a power-law localization. In equation (1), T is the transmission coefficient, $l_0 = 96E/w^2$ is the electron's localization length when $F = 0$ and $\langle\cdots\rangle$ denotes averaging over disorder.

Fewer details are known about the transmission coefficient properties in Q1D and two-dimensional (2D) disordered systems in an electric field [4, 12]. Kirkpatrik discussed Anderson localization and delocalization in a disordered system with one and two dimensions when an electric field is present [4]. Using a self-consistent diagrammatic theory of localization, his main conclusion was that in 2D systems Anderson localization is not possible in a finite electric field because the latter destroys the wave interference effects. In other words, Kirkpatrik found the existence of a mobility edge at some critical value of the electric field and impurity content or disorder. However, he did not succeed in properly defining a unique critical field separating the region of localized states ($F = 0$) from the region of possible extended states, and many details of such transitions, e.g. dependence on the number of propagating modes M, were left out from his analysis. What we are trying to point out is that a general analytical description of the localization length (LL) for the whole range of parameters F, L, M and E for a strip model is still lacking. This is why we addressed the problem and try to find some analytical results for the transmission properties (and hence for LL), in a weakly perturbed strip model with periodically distributed 2D δ functions with random strength. In our calculations we will also take into account an external longitudinal electric field. To the best of our knowledge, no such calculations have been previously reported.

This study is of interest in several respects. First of all, any analytical results for LL in a Q1D disordered system are highly desirable. Second, the dependence of LL on the energy across the energy bands of the pure system may be used to investigate changes in one-electron states in Q1D systems due to a change of the parameters of the system and due to an external electrical field. Finally, it is also relevant for experimental situations, e.g. for discussing the scaling of nonlinear (electric-field-dependent) resistivity in high-mobility Si-MOS structures as a function of electric field and electron density in both the metallic and insulating phases [13].

In particular we will consider a discrete lattice of size $(N \times S)$ with the system length $L = (N + 1)a_0$ and cross section $L_t = (S + 1)a_0$ ($L \gg L_t$, N, S to be integer numbers and a_0 is the spacing in between two potentials). In a further stage of our calculations we will assume also that $S = M$, i.e. we choose a discrete lattice whose points in the transverse direction coincide with the number of propagating modes M (see section 2.1).

It is worth noting that an external longitudinal electric field greatly complicates use of the transfer matrix method in Q1D or 2D systems to calculate scattering matrix elements. The complication mainly is connected with the fact that each individual transfer matrix is position-dependent and even after averaging over any disorder they remain distinct from one another and do not commute. However, the method proposed in [14, 15], allows for the solution of the problem analytically. With only minor modifications of the method, based on the Green's function (GF) formalism, we will show that in the approximation of white-noise potential in Q1D systems, an expression similar to equation (1) can be derived for LL. Hence one can find the critical value of the parameter X separating the exponentially localized states from the power-law states.

We have analyzed the ratio of the corresponding LL without and with electric field in the weak disordered regime. Somewhat expectedly, we found that the ratio collapses, for all modes M, into a universal curve in the Q1D strip model. This fact can be interpreted as an indirect indication of the applicability of the theory of single parameter scaling [16] (SPS) in Q1D systems and can be used to determine the critical regime of localization–delocalization transition.

Without pretending to give an exhaustive review, we briefly present some ideas regarding the SPS hypothesis for 1D and 2D disordered systems. As is well known, according to the SPS hypothesis all states are localized in 1D and the full conductance distribution function is described by a single parameter L/ξ, the ratio of the system size L to the localization length ξ. While for 1D systems the validity of the SPS hypothesis was carefully checked for many models in the weak localization regime and it has been proven to hold in most cases, the situation with SPS in Q1D and 2D systems is currently very controversial or poorly understood. In spite of the statement that all states in 2D are localized [16], there are a large number of experiments in which an apparent metal–insulator transition has been observed (see, e.g., [13, 17, 18] and references therein). On the one hand, careful numerical analysis [19, 20] of the 2D Anderson model showed excellent agreement with SPS while other studies [21, 22] suggested the existence of power-law localized states and two-parameter scaling. Nevertheless, the popular belief is that, in the limit of weak disorder, when the mean free path is macroscopic, SPS holds with arbitrarily high accuracy also in 2D systems of noninteracting electrons [23, 24].

We will restrict ourselves to a discussion of the weak disordered regime. This regime is of general interest since, when an external electric field is zero, localization occurs for an arbitrarily weak potential. In this case, LL can be calculated in the Born approximation for an uncorrelated potential in the Q1D systems without any restriction on the numbers of impurities (N_0) and modes (M) [14, 15]. For simplicity we will discuss the case when the number of modes M in the left and right leads connected to the Q1D system are the same. Furthermore, we assume that the external electric field is weak ($X < 1$).

This paper is organized as follows. In section 2 we briefly discuss the practical algorithm developed in [14, 15, 25] for solving the Dyson equation in the Q1D strip model with periodically arranged 2D δ impurities, with and without an external electric field. We will show that for such a system the scattering matrix elements can be described in terms of determinants of rank $N_0 \times N_0$. In section 2.2 we present results for the inverse localization length when an electric field is present. The results are discussed in section 3. The main conclusions are summarized in section 4.

2. Strip model with periodically arranged 2D δ impurities

First, we review the Dyson equation for a 2D strip model with impurity δ potential, (see equation (2)), studied in [14, 15, 25] when the external electric field is zero. This convenient formalism allows one to express the transmission coefficient of a wave propagating in a Q1D disordered structure through the determinant, which depends on the amplitudes of reflection of a single scatterer only. The rank of the characteristic determinant is $N_0 \times N_0$, as in 1D systems, and its zeros coincide with the poles of Q1D GF. Hence we can map the Q1D scattering problem into a 1D problem with modified matrix elements and obtain explicit results for LL in the Born approximation.

Consider the quantum transport of an electron in a Q1D disordered strip in the presence of N_0 2D Dirac δ potentials, distributed randomly on a strip

$$V(x, y) = \sum_{l=1}^{N_0} V_l \delta(x - x_l)\delta(y - y_l), \tag{2}$$

where (x_l, y_l) and V_l denote the position and the strength of the lth impurity in the (x, y) plane, respectively. V_l can be repulsive ($V_l > 0$) or attractive ($V_l < 0$).

The single-electron wavefunction is the solution of the Schrödinger equation

$$\left\{-\left(\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \frac{\mathrm{d}^2}{\mathrm{d}y^2}\right) + V_\mathrm{c}(y) + V(x, y) + U(x)\right\}\Psi(x, y) = E\Psi(x, y), \tag{3}$$

where the confinement potential $V_\mathrm{c}(y)$ depends only on the transverse direction y and $V(x, y)$ is the potential of the impurities in the Q1D strip given by equation (2). The potential $U(x) \equiv -Fx$ describes the external electric field.

The transverse-mode wavefunction $\chi_n(y)$ satisfies the 1D Schrödinger equation

$$\left\{-\frac{\mathrm{d}^2}{\mathrm{d}y^2} + V_\mathrm{c}(y)\right\}\chi_n(y) = E_n\chi_n(y), \tag{4}$$

where n is the subband index and E_n are the subband energies. In general, $\chi_n(y_l)$ depends on the choice of the confinement potential $V_\mathrm{c}(y)$. If we take $V_\mathrm{c}(y)$ to be zero for $0 \leqslant y \leqslant L_t$ and infinite elsewhere, then $\chi_n(y_l)$ is given by

$$\chi_n(y_l) = \sqrt{\frac{2}{L_t}}\sin\left(\frac{n\pi y_l}{L_t}\right). \tag{5}$$

The retarded GF for an electron traveling in the total potential satisfies the Schrödinger equation

$$\left[-\left(\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \frac{\mathrm{d}^2}{\mathrm{d}y^2}\right) + V_\mathrm{c}(y) + V(x, y) + U(x) - E\right] \times G^{(N_0)}(xy, x'y') = -\delta(x - x')\delta(y - y'). \tag{6}$$

The Dyson equation for a Q1D wire can be written in the form [26, 27]

$$G^{(N_0)}_{nm}(x, x') = G^{(0)}_n(x, x')\delta_{nm} + \sum_{k,q}\int G^{(0)}_n(x, x'')\delta_{nk}V_{kq}(x'')G^{(N_0)}_{qm}(x'', x')\,\mathrm{d}x'', \tag{7}$$

where Q1D GF $G^{(N_0)}_{nm}(x, x')$ is connected with retarded GF $G^{(N_0)}(xy, x'y')$ through the following relation:

$$G^{(N_0)}_{nm}(x, x') = \int \mathrm{d}y \int \mathrm{d}y'\chi^*_n(y)G^{(N_0)}(xy, x'y')\chi_n(y').$$

In equation (7) $G^{(0)}_n(x, x')$ is the GF in the absence of the defect potential $V(x, y)$ and obeys the equation

$$\left[-\frac{\mathrm{d}^2}{\mathrm{d}x^2} + U(x) - (E - E_n)\right]G^{(0)}_{nm}(x, x') = -\delta(x - x')\delta_{nm}. \tag{8}$$

Thus $G^{(0)}_{nm}(x, x')$ is diagonal in the indices n and m, i.e. $G^{(0)}_{nm}(x, x') = G^{(0)}_n(x, x')\delta_{nm}$. The upper index (l) of the GF (in equation (8) the index $l = 0$) indicates that the GF is calculated in the presence of $l\delta$ potentials.

Note that equation (7) is an infinite set of coupled algebraic equations which includes effects from both the open and closed modes, but in actual calculations we truncate the infinite set of coupled channels to a finite value M. This natural cutoff for the infinite series does not affect in any essential way the results discussed below [27, 28]. This is because the higher evanescent modes cannot be realized in a quantum wire: they will either dissipate or overcome the work function of the wire [29].

The matrix elements $V_{nm}(x)$ of the defect potential (2), after performing the integration over y, become

$$V_{nm}(x) = \int \chi^*_n(y)V(x, y)\chi_m(y)\,\mathrm{d}y = \sum_{l=1}^{N} V^{(l)}_{nm}\delta(x - x_l), \tag{9}$$

with the coupling constant $V^{(l)}_{nm}$ defined as

$$V^{(l)}_{nm}(y_l) = \chi^*_n(y_l)V_l\chi_m(y_)\equiv \frac{2V_l}{L_t}\sin\left(\frac{n\pi y_l}{L_t}\right)\sin\left(\frac{m\pi y_l}{L_t}\right). \tag{10}$$

The main algorithm for finding the GF for the whole system with $N_0\delta$ potentials is based on the idea of recursively building up the total GF. In such calculations, the GF is evaluated first when one δ potential is available. For this purpose we first isolate elements of the defect potential $V_{nm}(x)$ in the matrix, equation (9), the term corresponding to the last potential at x_{N_0}:

$$V_{nm}(x) = V^{(N_0)}_{nm}\delta(x - x_{N_0}) + \sum_{l=1}^{N_0-1} V^{(l)}_{nm}\delta(x - x_l). \tag{11}$$

We first evaluate the exact GF for a single δ potential. The case of two δ potentials, when we separate the next $(N_0 - 1)$th potential from the second term in expression (11), is then solved using the GF for a single δ potential. Then we solve the problem iteratively with $N_0\delta$ potentials by taking the solution with the $(N_0 - 1)$ known δs and extracting the scattering matrix elements. Thus we can obtain GF elements in an arbitrary interval $[x_n, x_{n+1}]$ $(n = 1, \ldots, N_0 - 1)$ of a disordered system.

In the following our main interest will be in the matrix elements of the GF for the range $x, x' \leqslant x_1$. This allows us to calculate the total transmission and reflection amplitudes of an electron which is incident on the system from the left. Using the well-known relations between the scattering amplitudes

and GF [30] the explicit form for the matrix elements of GF for $x, x' \leqslant x_1$ is

$$G_{nm}^{(N_0)}(x,x') = G_n^{(0)}(x,x')\delta_{nm} + R_{nm}^{(N_0)} \frac{G_n^0(x,x_1)G_m^0(x_1,x')}{\sqrt{G_n^0(x_1,x_1)G_m^0(x_1,x_1)}}. \tag{12}$$

Note that the expression (12) is valid for an arbitrary $G_n^{(0)}(x,x')$ (piece-wise constant potential, constant external electric and magnetic fields, etc). $R_{nm}^{(N_0)}$ is the matrix element of reflection amplitude from the whole system with $N_0\delta$ potentials, and may be written as the ratio of two determinants (for more details see [31]):

$$R_{nm}^{(N_0)} = (-1)^{N_0} \frac{1}{\det (D_{q,j}^{(N_0)})_{M,1}} \times \begin{vmatrix} 0 & r_{nm}^{(1)} & \cdots & r_{nm}^{(N_0)}(\lambda_n^{(1N_0)})^{1/2} \\ 1 & \cdots & \cdots & \cdots \\ \vdots & \vdots & & (D_{q,j}^{(N_0)})_{M,m} \\ (\lambda_m^{(1N_0)})^{1/2} & \vdots & & \end{vmatrix}, \tag{13}$$

where $r_{nm}^{(l)}$ are the complex reflection amplitudes from the isolated potential V_l in the absence of the remaining $(N_0 - 1)$ potentials [27]

$$r_{nm}^{(l)} = \frac{V_{nm}^{(l)} \sqrt{G_n^{(0)}(x_l,x_l)G_m^{(0)}(x_l,x_l)}}{1 - \sum_p^M V_{pp}^{(l)} G_p^{(0)}(x_l,x_l)}. \tag{14}$$

Note that $r_{nm}^{(l)}$ satisfy the identity $r_{mm}^{(l)} r_{nn}^{(l)} - r_{mn}^{(l)} r_{nm}^{(l)} = 0$, which can be checked directly, making use of equation (14).

Using the relation [31] connecting $G(x,x')$ with the one-particle GF at coinciding one-dimensional coordinates $x = x'$, the quantity $\lambda_n^{(jq)}$ in equation (13) can be presented in the form

$$\lambda_n^{(qj)} = \lambda_n^{(jq)} = \frac{G_n^{(0)}(x_j,x_q)G_n^{(0)}(x_q,x_j)}{G_n^{(0)}(x_j,x_j)G_n^{(0)}(x_q,x_q)} \equiv \exp\left[\int_{\min(x_q,x_j)}^{\max(x_q,x_j)} \frac{\mathrm{d}x}{G_n^{(0)}(x,x)}\right] \tag{15}$$

where $\int \frac{\mathrm{d}x}{G_n^{(0)}(x,x)}$ is the phase that an electron acquires during its motion in the field $U(x)$ between the scatterers j and q.

The numerator of $R_{nm}^{(N_0)}$ is obtained from the quantity $\det (D_{q,j}^{(N_0)})_{M,m}$ by augmenting it on the left and on the top. The matrix elements of the denominator $(D_{q,j}^{(N_0)})_{M,m}$, which contains information about the number of modes $M[1 \leqslant q, j \leqslant N_0; 1 \leqslant m \leqslant M]$, are

$$(D_{q,j}^{(N_0)})_{M,m} = -\delta_{qj} + (1-\delta_{qj}) \sum_{p=1}^{M} \frac{r_{1p}^{(q)} r_{pm}^{(j)}}{r_{1m}^{(q)}} (\lambda_p^{(qj)})^{1/2}. \tag{16}$$

The determinant $(D_{q,j}^{(N_0)})_{M,m}$ is in general a complex function of the energy E. The quasibound states of an electron in the disordered Q1D or in a 2D system correspond to the poles of GF for the whole system, which coincide with the zeros of $\det(D_{q,j}^{(N_0)})_{M,1}$. Therefore, to obtain the bound energy spectrum numerically, we must calculate $\det(D_{q,j}^{(N_0)})_{M,1}$ and find simultaneous zeros of its real and imaginary parts. A specific case of this equation, the spectrum of the single δ-function scatterer ($N_0 = 1$) with finite number of modes M in a Q1D waveguide (when $F = 0$), was studied in [32]. Note that equation (16) reduces to the characteristic determinant of a purely 1D system (see [33, 34]) if there is no coupling to the second, third, etc, modes, i.e. $r_{p1}^{(p)} = r_{1p}^{(p)} = 0$.

Putting in the appropriate GF matrix elements, equation (12), $x = x_1$ and $x' = x_{N_0}$, one can calculate the transmission amplitude $T_{nm}^{(N_0)}$ of an electron through the system with $N_0\delta$ potentials. Similarly to $R_{nm}^{(N_0)}$, we can write the explicit form for $T_{nm}^{(N_0)}$ as a ratio of two determinants [14, 15]:

$$T_{nm}^{(N_0)} = (-1)^{N_0} \frac{(\lambda_m^{(1N_0)})^{1/2}}{\det (D_{q,j}^{(N_0)})_{M,1}} \times \begin{vmatrix} \delta_{nm} & r_{nm}^{(1)} & \cdots & r_{nm}^{(N_0)}(\lambda_n^{(1N_0)})^{1/2} \\ 1 & \cdots & \cdots & \cdots \\ \vdots & \vdots & & (D_{q,j}^{(N_0)})_{M,m} \\ (\lambda_m^{(1N_0)})^{-1/2} & \vdots & & \end{vmatrix}, \tag{17}$$

where the numerator is obtained from the same determinant (16) by augmenting it on the left and on the top.

Using equations (13) and (17), it is straightforward to check by mathematical induction that, for the scattering matrix elements, current conservation takes place:

$$\sum_{m=1}^{M} (T_{nm}^{(N_0)} T_{nm}^{(N_0)*} + R_{nm}^{(N_0)} R_{nm}^{(N_0)*}) = 1, \tag{18}$$

where the summation is carried out over the propagating modes only.

2.1. Zero external electric field

The case of zero electric field was discussed in [14, 15]. For $U(x) \equiv 0$, the explicit form of $G_n^{(0)}(x,x')$ which satisfies equation (8) and corresponds to a propagating mode is

$$G_n^0(x,x') = -\frac{\mathrm{i}}{2k_{n,0}} \mathrm{e}^{\mathrm{i}k_{n,0}|x-x'|}, \tag{19}$$

with wavevector

$$k_{n,0} = +\sqrt{E - \frac{n^2\pi^2}{L_t^2}}, \tag{20}$$

where the subscript index 0 indicates zero electric field.

Thus we have from equation (15)

$$\lambda_n^{(qj)} = \mathrm{e}^{2\mathrm{i}k_{n,0}|x_q - x_j|}. \tag{21}$$

If $E < E_n$ and mode n is an evanescent mode, then in equations (19) and (21) one must take an analytic continuation of the $k_{n,0} = \mathrm{i}\kappa_{n,0}$, where $\kappa_{n,0} = +\sqrt{\frac{n^2\pi^2}{L_t^2} - E}$.

In our further calculations we evaluate the scattering matrix elements $T_{nm}^{(N_0)}$, equation (17), in the weak disorder regime. This means that it is sufficient to restrict the expansion of the expressions of $T_{nm}^{(N_0)}$ to first order in the δ-potential amplitude. In other words, in the determinants we only kept the terms that are proportional to $r_{nm}^{(l)}$—the complex reflection

amplitude (14) from the isolated potential V_l. The result for the electron's transmission amplitude $T_{nm}^{(N_0)}$ is

$$T_{nm}^{(N_0)} \approx \mathrm{e}^{\mathrm{i}k_{m,0}(x_{N_0}-x_1)} \times \begin{cases} \dfrac{1}{1+\mathrm{i}\sum_{l=1}^{N_0} V_l A_l}\left[1+\mathrm{i}\sum_{l=1}^{N_0} V_l\left(A_l - \dfrac{\sin^2(\frac{m\pi y_l}{L_t})}{k_{m,0}L_t}\right)\right] \\ \qquad \text{if } n=m, \\ \dfrac{-\mathrm{i}}{L_t\sqrt{k_{n,0}k_{m,0}}}\sum_{l=1}^{N_0} V_l \sin\left(\dfrac{n\pi y_l}{L_t}\right) \\ \times \sin\left(\dfrac{m\pi y_l}{L_t}\right)\mathrm{e}^{\mathrm{i}(k_{n,0}-k_{m,0})(x_l-x_1)} \qquad \text{if } n\neq m, \end{cases} \tag{22}$$

with $A_l = \sum_{n=1}^{M} \frac{\sin^2(n\pi y_l/L_t)}{L_t k_{n,0}}$. y_l is the coordinate of the lth δ impurity in the y direction, and x_1 and x_{N_0} are the x coordinates of the first and last (N_0^{th}) δ functions. The wavenumbers $k_{n,0}$ for the propagating modes are defined by equation (20). A finite number M includes the effects from both propagating and evanescent modes. The wavenumbers of the evanescent modes are obtained by setting $k_{n,0} = \mathrm{i}\kappa_{n,0}$.

As noted in [15, 35], in Q1D systems the phase in the transmission amplitude, $T_{nm}^{(N_0)}$, is irrelevant for a white-noise potential where $\langle V_i V_k\rangle = \alpha\delta_{ik}$. In other words, the configuration of δ potentials is not important for uncorrelated potentials in the linear approximation of perturbation theory. This means that, without any loss of generality, in the x direction we can arrange δ functions periodically with spacing $a_0 = 1$ and thus replace the finite length $L = (N+1)a_0$ by N ($N \gg 1$).

As a further simplification we focus on the case of periodically arranged 2D δ-potential scatterers on the lattice points of a strip and discuss the case when the spacing between scatterers is constant and equal in the x and y directions and of unit length. This means that in the y direction we choose a discrete lattice whose points are located at $y_l = l$, ($l = 1, 2, \ldots, S$) and thus the width of the wire $L_t = S + 1$. Now let S coincide with the number of propagating modes M, i.e. we will replace $L_t \to M + 1$. This is an essential point for our further calculations because it allows us to consider the 2D δ-function strip model as a discrete lattice of size $(N \times M)$ (instead of $(N \times S)$), where N and M are the number of δ potentials in the x and y directions, respectively.

Recalling that the number of open modes $M = \mathrm{Int}(\sqrt{E}L_t/\pi)$ (see equation (20)), i.e. the integer part of $\sqrt{E}L_t/\pi$, and replacing $L_t = M + 1$ one can show that the $\sqrt{E}$ range is defined by

$$\frac{\pi M}{(M+1)} \leqslant \sqrt{E} \leqslant \frac{\pi(M+1)}{(M+2)}. \tag{23}$$

For energy E lying in this range the first M modes or channels can propagate along the Q1D system. In our further numerical calculations we will choose the energy E according to this inequality.

Using the known relation for the inverse localization length ξ_M as a function of the system size L and modes M

$$\xi_M^{-1} = -\lim_{L\to\infty}\frac{1}{2ML}\left\langle \ln\sum_{n,m}^{M}|T_{nm}^{(N)}|^2\right\rangle, \tag{24}$$

and after ensemble-averaging over the random potentials V_l distributed uniformly in an interval $[-w/2, w/2]$ with the use of explicit expressions of $T_{nm}^{(N)}$, equation (22), we arrive at the following expression for the inverse localization length [14, 15]:

$$\frac{1}{\xi_M^0} = \frac{\alpha}{16M^2(M+1)} \times\left[\sum_{n=1}^{M}\frac{3+\delta_{2n,M+1}}{k_{n,0}^2} + 2\sum_{n<m}^{M}\frac{2+\delta_{n+m,M+1}}{k_{n,0}k_{m,0}}\right], \tag{25}$$

with $\alpha = \langle V_l^2\rangle = w^2/12$.

To close this subsection we note that in equation (25) one can say formally that all k_n are equal, if there is no coupling to the second, third, etc, modes, i.e. $r_{p1}^{(p)} = r_{1p}^{(p)} = 0$. Then, after the summation over the modes (which will give $2M(M+1)$) we find as a result $\xi_M^0 = M\xi_1^0$; $\xi_1^0 = 8/\alpha^2$ denotes the localization length of a purely 1D system. As is clear the found result is somewhat expected: it confirms the prediction of Thouless [36] that in the limit of weak coupling ξ_M^0 must be proportional to M. In this sense the $1/\xi_M^0$, equation (25), is smaller by an additional factor $1/M$ from the analogous expression (19) of [14]. The reason was that in [14] we used the definition of inverse localization length which differs from (24) by the same factor $1/M$.

2.2. Inclusion of external electric field

The exact solution to equation (3) with the external electric field $U(x) = -Fx$ is Airy-function-like with

$$k_{n,x} = \sqrt{E - n^2\pi^2/L_t^2 + Fx}.$$

For weak fields ($Fa_0 < E$, where a_0 is the spacing between two δ potentials) one can use the so-called ladder approximation. This means that instead of the Airy functions we use the plane waves (see, e.g., [2, 3, 9]) and use the Poincaré map representation of the Schrödinger equation.

Proceeding along the same lines as in section 2.1 we can show that only minor modifications of the final expression for LL in equation (25) are required in order to include the weak electric field. As a result of the ladder approximation, as well as the fact that for a white-noise potential the phase of the transmission amplitude is irrelevant, we obtain the inverse localization length which explicitly depends on the coordinate x:

$$\frac{1}{\xi_M^F(x)} = \frac{\alpha}{16M^2(M+1)} \times\left[\sum_{n=1}^{M}\frac{3+\delta_{2n,M+1}}{k_{n,x}^2} + 2\sum_{n<m}^{M}\frac{2+\delta_{n+m,M+1}}{k_{n,x}k_{m,x}}\right]. \tag{26}$$

For convenience, we introduce the superscript F (meaning that an electric field is present) in order to facilitate the comparison with the corresponding expression when $F = 0$. Since we are considering many potentials and the $-Fx$ potential changes slowly, we can integrate $1/\xi_M^F(x)$ over the length L of the

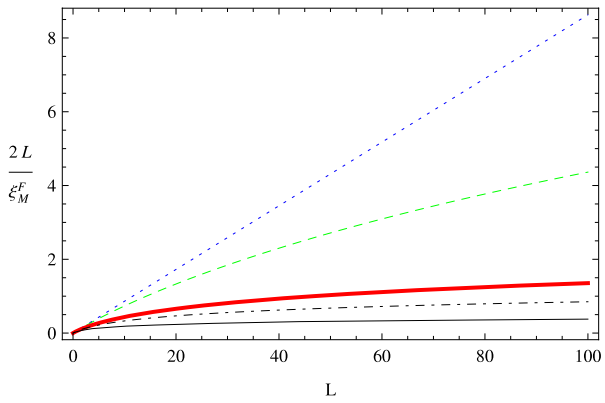

Figure 1. Plot of the inverse localization length $2L/\xi_M^F(L)$ versus L for different electric fields F. For all curves the number of modes $M = 15$, energy $E = 0.9\pi^2$ and disorder $w = 0.5$. The case of $F = 0$ is represented by the straight dotted line, based on equation (25), indicating exponential localization. The dashed, thick, dotted–dashed solid curves correspond to $F = 0.05$, $F = 0.5$, $F = 1$ and $F = 3$, respectively.

system. This leads us to the following expression for the inverse LL:

$$\frac{1}{\xi_M^F(L)} \equiv \frac{1}{L}\int_0^L \frac{\mathrm{d}x}{\xi_M^F(x)} = \frac{\alpha}{16M^2(M+1)EX} \times \left[\sum_{n=1}^{M}(3+\delta_{2n,M+1})\ln\left(1+\frac{EX}{k_{n,0}^2}\right) + 2\sum_{n<m}^{M}(2+\delta_{n+m,M+1})\ln\left(1+\frac{EX}{k_{n,0}k_{m,0}}\right)\right] \quad (27)$$

where we remember that $X \equiv FL/E$ and $k_{n,0}$ is given by equation (20).

Equation (27) represents the central result of this work. It express the LL in terms of the number of modes M, disorder w, incoming energy E and electric field F. For the single channel case, $M = 1$, equation (27) reduces to equation (1), as it should. In the limit $F \to 0$, one correctly recovers equation (25) discussed in [14, 15]. In the following section we analyze its limits.

3. Results

Let us start our discussion of the results from the weakly localized regime, i.e. $X \ll 1$. The behavior of $1/\xi_M^F(L)$ in this limit can be more easily obtained from the series expansion for $\ln(1+x)$. Expanding up to order x^2 we get

$$\frac{1}{\xi_M^F(L)} \approx \frac{1}{\xi_M^0} - \frac{FL}{2}f(M), \quad (28)$$

where $1/\xi_M^0$ was defined in equation (25) and $f(M)$ has the form

$$f(M) = \frac{\alpha}{16M^2(M+1)}\left[\sum_{n=1}^{M}\frac{3+\delta_{2n,M+1}}{k_{n,0}^4} + 2\sum_{n<m}^{M}\frac{2+\delta_{n+m,M+1}}{k_{n,0}^2k_{m,0}^2}\right].$$

By replacing sums by integrals, performing them and keeping only the relevant terms, after some algebra, the following asymptotic formula for inverse LL is obtained in the thermodynamic limit ($M \to \infty$):

$$\frac{1}{\xi_{M\to\infty}^F} \approx \frac{\alpha}{32M\pi^2}\left\{\pi^2 + \frac{\ln(2M)}{M} - \frac{FL}{2\pi^2}\left[1+\ln^2(2M)+\frac{\ln(2M)}{M}\right]\right\}. \quad (29)$$

The important feature of equations (28) and (29) is that they already indicate that even for small electric field the states become less localized.

To look at the influence of the electric field, we plot $2L/\xi_M^F(L)$, based on equation (27), with electric field and without, versus the system length L for fixed values of the disorder $w = 0.5$ and number of channels $M = 15$ (see figure 1 caption for details). The incoming energy $E = 0.9\pi^2$ is chosen from the range (23). The case of zero electric field, $2L/\xi_M^{(0)}$, is presented as a dotted straight line, based on equation (25), indicating exponential localization. It can be seen that $2L/\xi_M^F(L)$ follows a straight line for small L and electric field F. The approximate expression (28) describes this behavior well. With further increase of L the curve starts to bend, increase slowly and finally tends to a constant value. All the curves vary qualitatively in the same way and are quite similar to the behavior of the LL in 1D disordered systems in a constant electric field F, discussed in [1–3, 7]. Hence it is not surprising that the curves in figure 1 generally capture some feature of the 1D case, i.e. change from straight to flat when the electrostatic energy FL is approximately equal to the electron's incoming energy E, when $X = FL/E$ is of the order of unity. For $X \ll 1$ the kinetic energy gained by the electron from the field is small compared to E and, therefore, the random potential is dominant and states are exponentially localized. This corresponds to the very left part of figure 1 for L smaller than 30. For very large X, i.e. when the total electrostatic energy is much larger than the energy of the particle, the potential becomes a small perturbation and states are delocalized. It is worth noting that, as in the 1D case [1–3, 7], the saturation value of $2L/\xi_M^F(L)$ for large L varies drastically with even small changes in the electric field F.

We show the dependence of $2L/\xi_M^F(L)$ versus $\log L$ for large L in figure 2 for the same parameter values as in figure 1. Notice that the straight lines have different slopes: the slopes are smaller the larger F is. Such a linear behavior of $2L/\xi_M^F(L)$ versus $\log L$ means that in Q1D systems states are weakly localized, i.e. we deal with power-law localization (for more details see [1–3, 7], where a transition from exponential to power-law behavior with increasing electric field in 1D mesoscopic systems is discussed).

In figure 3 we have plotted $2L/\xi_M^F(L)$ as a function of the system length L for a different propagating mode M. The appropriate values of E are chosen from inequality (23). The values of the disorder $w = 0.5$ and the electric field $F = 0.01$ are fixed. We see from this figure that increasing the number of modes M, for fixed electric field F, slightly enhances the localization length. In other words, the electron states of the

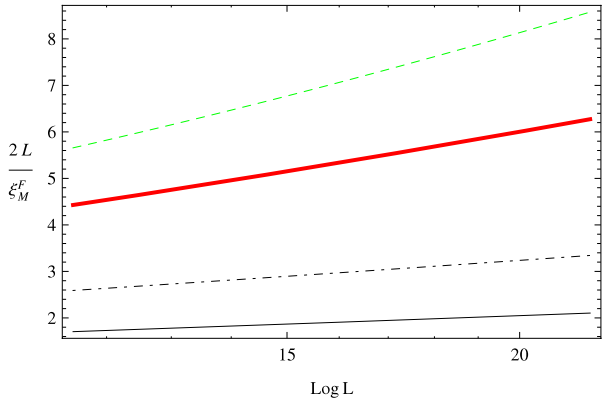

Figure 2. $1/\xi_M^F(L)$ versus $\log L$ with the same parameter values as in figure 1.

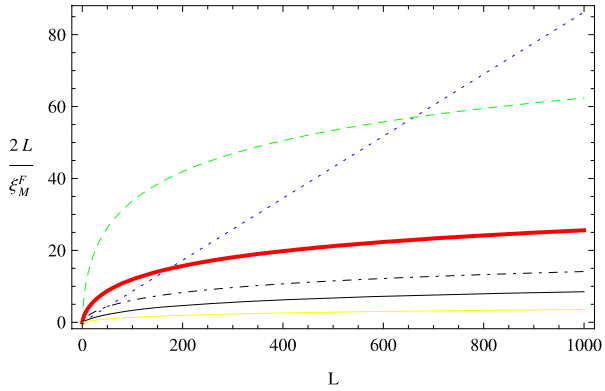

Figure 3. Plot of the inverse localization length $2L/\xi_M^F(L)$ versus L for different numbers of modes M, energy E and for disorder $w = 0.5$. For all the curves the electric field $F = 0.01$. The case of $F = 0$ is represented by the straight dotted line, indicating exponential localization ($M = 15$, $E = 0.9\pi^2$). The dashed, thick, dotted–dashed, thin unbroken and solid curves correspond to ($M = 3$, $E = 0.568\pi^2$), ($M = 4$, $E = 0.65\pi^2$), ($M = 6$, $E = 0.74\pi^2$), ($M = 15$, $E = 0.9\pi^2$) and ($M = 35$, $E = 0.95\pi^2$), respectively.

system are less localized for large M. From the comparison of figures 2 and 3 one can see that the curves change significantly with increasing F (for fixed M) rather than with increasing M (for fixed F). In order to verify this fact and get a quantitative answer, at least for small F, we have calculated the partial derivatives of $1/\xi^F_{M\to\infty}$, equation (29), with respect to F and M. The ratio of these derivatives leads us to the expression of the form

$$\left|\frac{\partial_F(1/\xi^F_{M\to\infty})}{\partial_M(1/\xi^F_{M\to\infty})}\right| \approx LM\ln^2(2M)[1+O(M^{-1})]. \tag{30}$$

As is clear from equation (30) the ratio is always larger than one, which is a manifestation of the fact that a change of F affects the LL more effectively than M. Note that our numerical calculations, based on equation (27), confirm this statement for an arbitrary M and F.

In the opposite limit of very large $X \gg 1$, equation (27) yields

$$\frac{1}{\xi_M^F(L)} \approx \frac{\alpha}{8M\pi^2X}\left[\ln(\pi^2X) - \frac{2M+1}{2M(M+1)}\sum_{n=1}^{M}\ln k_{n,o}^2\right]. \tag{31}$$

For large M summation can be replaced by integration, yielding the logarithmical asymptotic behavior of $1/\xi_M^F$:

$$\frac{1}{\xi_M^F} \approx \frac{\alpha}{8M\pi^2X}\left[\ln X + 2\ln\frac{e}{2}\right] + O(M^{-2}). \tag{32}$$

Note that, following equations (29) and (32), the asymptotic behavior of the ratio ξ_M^0/ξ_M^F in the limit of large M, can be presented:

$$\frac{\xi_M^0}{\xi_M^F} \approx \frac{4}{\pi^2X}\left[\ln X + 2\ln\frac{e}{2}\right] + O\big(M^{-1}\big). \tag{33}$$

Also note that in a 1D system the analogous ratio, equation (1), in the same limit of $X \gg 1$, behaves as $\approx \ln X/X$.

In figure 4 we plot ξ_M^0/ξ_M^F, the ratio of the corresponding localization length without electric field, equation (25), to the localization length when an electric field is present, equation (27), versus $X = FL/E$. The different values of the propagating mode and the appropriate energy are specified in

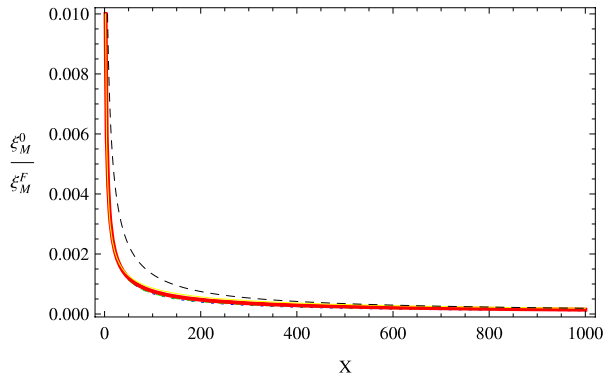

Figure 4. Plot of the ratio $\xi_M^0/\xi_M^F(L)$ versus $X = FL/E_F$ for different numbers of modes $M = 4, 6, 10, 15, 35$ and appropriate energies: $E = 0.65\pi^2, 0.74\pi^2, 0.83\pi^2, 0.9\pi^2$, $E = 0.95\pi^2$. Disorder $w = 0.5$. and electric field $F = 0.1$. The dashed line shows the approximate expression (33).

the figure caption. The values of the disorder and the electric field are $w = 0.5$ and $F = 0.1$, respectively. As we can see from figure 4 the curves collapse into one universal curve in the Q1D strip model for all modes M. In the same figure with a dashed curve we have presented the field-dependent approximate expression (33). The far tail of the universal curve can be fitted fairly well with this expression. Surprisingly, very good agreement is obtained even for relatively small values of X. The good overlap of the curves indicates the validity of the SPS hypothesis in a Q1D disordered system, although the definite answer concerning the applicability of SPS theory can only give us a careful analytical and numerical analysis of the statistical properties [13, 17, 18].

4. Conclusion

A two-dimensional δ-potential Kronig–Penney model for quasi-one-dimensional disordered systems is used to study

analytically the influence of a constant electric field on the inverse LL. Based on Green's function poles we have calculated the inverse LL as a function of the incoming energy E, field F, length L of the Q1D sample, number of modes M in the transverse direction and the measure of disorder w. We show that for fixed values of the channel M and for the finite electric field F the LL $1/\xi_M^F(L)$ follows a straight line for small L, but with increasing L the graph curve starts to bend, increases slowly and finally tends to a constant value. This means that, for large L in Q1D systems, states are weakly localized, i.e. we deal with power-law localization. In other words, a transition from exponential to power-law behavior takes place with increasing electric field in Q1D mesoscopic systems. We noted that the graphs change significantly with increasing F (for fixed M) rather than with increasing M (for fixed F). To argue this empirical observation, we have calculated analytically the partial derivatives of $1/\xi_{M\to\infty}^F$, equation (29), with respect to F and M. The ratio of these derivatives is always larger than one. We have shown that all the curves representing the ratio of the corresponding localization length with and without electric field collapse, for all modes M and an energy E, into a universal curve in the Q1D strip model. The far tail of the universal curve can be fitted fairly well with the expression (33). The good overlap of the curves indicates the validity of the SPS hypothesis in a Q1D disordered system, although the definite answer concerning the applicability of SPS theory can only be found through careful analytical and numerical analysis of the statistical properties [13, 17, 18].

We expect that the present results may motivate further work along these lines.

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