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Oscillatory Current along a Tunnel-Thin Potential Barrier in a Magnetic Field

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Investigations of the electron energy spectrum near the surface in the presence of a homogeneous magnetic field parallel to the surface are usually carried out assuming boundary conditions for the wave function on the surface to be zero, which is equivalent to the electron wave total reflection condition at the surface. In heterostructures when a potential barrier at the border is present, or in a homogeneous semiconductor system when a tunnel-thin dielectric is present, the electron reflection amplitude (r) can vary within a broad range in the interval $0 \leqq |r| \leqq 1$ and r can be essentially depending on energy. Let us direct the magnetic field along the z axis, lying in the barrier plane (z and y parallel to the barrier, x perpendicular to the barrier) and choose the vector potential in the form $\vec{A} = (0, x H, 0)$. It is also interesting to observe how the Landau levels degenerated with respect to the impulse component P_y transfer into surface magnetic states, when degeneracy with respect to P_y vanishes and all states between Landau levels are filled (at a fixed value of the impulse component along the magnetic field) as the barrier penetrability decreases. As we shall show, forbidden and allowed narrow bands, so-called mini-bands, appear in the electron energy spectrum in the presence of a semi-penetrable barrier in a magnetic field parallel to the barrier.

We shall consider the case when in the system there is a thin potential barrier, for instance, a tunnel-thin dielectric in a homogeneous semiconducting system. If the thickness of the dielectric layer is smaller than the magnetic length, one can pass to the approximation of a δ-like potential. Then the Schrödinger equation (for simplicity we neglect here spin splitting of the levels in the magnetic field) may be written in the form

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$$\left[\frac{1}{2m}\left(\vec{P} - \frac{e}{c}\vec{A}\right)^2 + V\,\delta(x)\right]\Psi(\vec{r}) = E\,\Psi(\vec{r})\ . \tag{1}$$

The solution of (1) may be presented in the form /1/

$$\Psi(\vec{r}) = C' \exp\left\{i k_y y + i k_z z\right\} G(\xi, \xi_0; \nu)\ , \tag{2}$$

where

$$\xi = \sqrt{m\omega/\hbar}(x - x_0), \quad x_0 = P_y/m\omega, \quad \omega = eH/mc,$$
$$E' = E - (P_z^2/2m) = \hbar\omega(\nu + \tfrac{1}{2})\ ,$$

$G(\xi, \xi'; \nu)$ is the electron Green function (GF) in the magnetic field (the potential barrier being absent).

The electron energy spectrum in the presence of the barrier is determined by the equation (see /1/)

$$G(\xi_0, \xi_0; \nu) = V^{-1}\ . \tag{3}$$

Thus the problem is brought to the finding of the GF in explicit form.

The calculation of the explicit form of the electron GF in the magnetic field has become subject of many studies in various forms. It is necessary for us to determine the explicit form of this function in one-dimensional coordinate and energy representations. It is simpler to calculate it from the formula

$$G(\xi, \xi'; \nu) = C\left[\Psi_1(\xi)\Psi_2(\xi')\,\Theta\,(\xi - \xi') + \Psi_1(\xi')\Psi_2(\xi)\,\Theta\,(\xi' - \xi)\right]\ , \tag{4}$$

where Θ is the step function, Ψ_1 and Ψ_2 are the two solutions of the corresponding one-dimensional Schrödinger equation satisfying the conditions $\Psi_1(\xi \to \infty) \to 0$ and $\Psi_2(\xi \to -\infty) \to 0$, C is the normalizing coefficient determined by the condition

$$C = \left[\Psi_1'\Psi_2 - \Psi_1\Psi_2'\right]^{-1}_{\xi = \xi'}\ .$$

The functions Ψ_1 and Ψ_2 are of the form

$$\Psi_{1,2}(\xi) = e^{-\xi^2/2}\left[\Gamma^{-1}\left(\frac{1-\nu}{2}\right)\Phi\left(-\frac{\nu}{2}, \frac{1}{2}; \xi^2\right) + \Gamma^{-1}\left(-\frac{\nu}{2}\right)2\xi\,\Phi\left(\frac{1-\nu}{2}, \frac{3}{2}, \xi^2\right)\right], \tag{5}$$

where Γ is the Gamma function, Φ is the degenerated hypergeometric function.

The final expression for the GF is of the form (4), with functions (5), where

$$C = \frac{1}{4}\Gamma\left(-\frac{\nu}{2}\right)\Gamma\left(\frac{1-\nu}{2}\right)\ .$$

Short Notes

In quasi-classical ap
simple formula

$$G(\xi, \xi'; \nu) = -$$

Taking into account
($\lambda = \sqrt{\hbar/m\omega}$)

$$\sin \pi\nu -$$

From this it is easy
we have $\nu = 0, 1, 2, .$
$\nu = 1, 3, \ldots .$

The analysis of (7)
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In quasi-classical approximation $\nu \gg 1$, at ξ, $\xi' \ll \sqrt{2\nu+1}$ we have the simple formula

$$G(\xi,\xi';\nu) = -\frac{m\lambda}{\hbar^2\sqrt{2\nu+1}\sin\pi\nu}\left[\cos(\pi\nu - |\xi-\xi'|\sqrt{2\nu+1}) + \cos(\xi+\xi')\sqrt{2\nu+1}\right]. \quad (6)$$

Taking into account (6), equation (3) will be written in the form ($\lambda = \sqrt{\hbar/m\omega}$)

$$\sin\pi\nu - \frac{mV\lambda}{\hbar^2\sqrt{2\nu+1}}(1+\cos\pi\nu) = -\frac{2mV\lambda}{\hbar^2\sqrt{2\nu+1}}\sin^2\frac{x_0}{\lambda}\sqrt{2\nu+1}. \quad (7)$$

From this it is easy to get the known results in asymptotic cases: at $V \to 0$, we have $\nu = 0, 1, 2, \ldots$ (Landau levels); at $V \to \infty$ and $x_0 \to 0$, we have $\nu = 1, 3, \ldots$.

The analysis of (7) at $0 < V < \infty$ shows that the energy spectrum of the electron depending on P_y gets band character - allowed and forbidden mini-bands appear. The energy, considered in quasi-classical approximation, is periodically dependent on P_y, the period $2\pi\lambda m\omega(2\nu+1)^{-1/2}$ itself depends on the level number of volume quantization $\nu = n$; $n = 0, 1, 2, \ldots$.

In a sufficiently thin barrier case $2mV\lambda\hbar^{-2} \ll \sqrt{2\nu+1}$ narrow ($|\nu - n| \ll 1$) allowed and relatively wide forbidden mini-bands appear. The dispersion law in this case has the form

$$\nu = n + \frac{mV\lambda}{\hbar^2\pi\sqrt{2n+1}}\left[1 + \cos\left(\pi n + \frac{x_0}{\lambda}\sqrt{2n+1}\right)\right]. \quad (8)$$

Naturally the obtained simple analytical formulas for the GF (6) and dispersion law (8) are true, when the above-mentioned assumptions are made. Nevertheless the conclusion about the formation of allowed and forbidden mini-bands is always true, if only $0 < V < \infty$, of which one may be convinced by graphical solution of (3) with the exact form of the GF (4).

In conclusion let us note that the formation of a mini-band structure of the electron energy spectrum may appear, influencing, for instance, the current along the barrier and perpendicular to the magnetic field direction, when there is an additional electric field $\mathcal{E} = \mathcal{E}_y$. By analogy with the case considered in /2/ for a superlattice, here the current depending on $\mathcal{E}_y$ will be of oscillatory character. For experimental tests, the required conditions (at low temperature, $T < 5$ K, strong magnetic field of $B \approx 10$ T) are available and are used in investigations of the quantum Hall effect.

References

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3d-Impurity Level

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$$\sum_{\nu} C_{\nu} (H_{\mu\nu} - E_i G_{\mu}$$

where C_{ν} are the coeffi
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on the semi-empirical M
proximation, $H_{\mu\nu}$ can b

$$H_{\mu\mu} = -I_{\mu} \; ; \quad H_{\mu\nu}$$

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