CALIFORNIA STATE UNIVERSITY, BAKERSFIELD
Lee Webb Math Field Day 2025
Team Medley, Junior-Senior Level

- Each correct answer is worth ten points. Answers require justification. Partial credit may be given. Unanswered questions are given zero points.
- You have 50 minutes to complete the Exam. When the exam is over, give only one set of answers per team to the proctor. Multiple solutions to the same problem will invalidate each other.
- Elegance of solutions may affect score and may be used to break ties.
- All calculators, cell phones, music players, and other electronic devices should be put away in backpacks, purses, pockets, etc. Leaving early or otherwise disrupting other contestants may be cause for disqualification.

1. The arithmetic derivative $D(n)$ of a natural number n is defined as $D(p) = 1$ if $p = 1$ or p is prime, and $D(ab) = aD(b) + bD(a)$ for any natural numbers a, b. Find

$$\frac{D(20^{25})}{20^{25}}$$

2. Consider the number of distinct permutations of the characters

T O D A Y I S M A T H F I E L D D A Y

What digit appears right before the trailing zeros at the end?

3. A standard deck of cards is shuffled and dealt into 4 equal piles. What is the probability there is a king in each pile? You may leave your answer as the product and quotient of integers.

4. What value(s) of x minimize the expression

$$|x-1| + |x-2| + |x-2^2| + |x-2^3| + |x-2^4| + \cdots + |x-2^{10}|$$