CALIFORNIA STATE UNIVERSITY, BAKERSFIELD
Lee Webb Math Field Day 2024
Individual Medley, Freshman- Sophomore Level

WITH ANSWERS

Your answers to these questions should be on the side of the answer sheet that has answer spaces 1, 2, 3, 4, 5 (NOT A, B, C, D, E). On the answer sheet you should write your name, school name, level (Freshman-Sophomore), and Division (your proctor should have a list of which schools are in which divisions).

For each of the following questions, blacken the appropriate circle on the answer sheet. Each correct answer is worth four points. **One point is deducted for each incorrect answer.** An unanswered question is given zero points. Note that random guessing may adversely affect your score.

You have 50 minutes to complete the examination. If you finish early, review your answers. When the exam is over, give your answer sheet to the proctor.

All calculators, cell phones, music players, and other electronic devices should be put away in backpacks, purses, pockets, etc. Leaving early or otherwise disrupting other contestants may be cause for disqualification.

1. In a certain meeting, every participant shakes hands with every other. All together there were 45 handshakes. How many participants are in the meeting?

1. 90
2. 45
3. 15
4. 10 ⌘
5. 8

2. Circle C has area equal to 8π . What is the area of an equilateral triangle inscribed in C?

1. 6
2. $6\sqrt{2}$
3. $6\sqrt{3}$ ⌘
4. $6\sqrt{6}$
5. 8

3. The least common multiple of 360 and some number b is 5040 (= 16x9x35). The greatest common divisor of the same two numbers is 24. What is the value of *b* ?

1. 144
2. 282
3. 300
4. 336 ⌘
5. 420

4. Simplify this compound fraction:

$$\frac{1-\frac{5}{8}}{1-\frac{5}{1-\frac{5}{8}}}$$

1. $\frac{-9}{296}$ ⌘
2. $\frac{16}{257}$
3. $\frac{51}{218}$
4. $\frac{128}{91}$
5. $\frac{-151}{228}$

5. How many distinct solutions does the equation $x^4 - x^3 = -x^2/4$ have?

1. 0 2. 1 3. 2 ⌘
4. 3 5. 4

6. Which of the following numbers is the largest?

1. 10^{100} 2. 20^{80} ⌘ 3. 30^{60}
4. 40^{40} 5. 50^{50}

7. Suppose we know that $-2 < x < 5$, then $|x+2| + |x-5|$ equals

1. $2x-3$ 2. 7 ⌘ 3. $7-x$
4. $3-2x$ 5. other

8. Suppose *F(x)* is a function that satisfies *F(xy)=F(x)+F(y) -1* for all real numbers *x* and *y*. Also assume that *F(2) = 3*. Determine *F(8)*.

1. 4 2. 5 3. 6
4. 7 ⌘ 5. 8

9. Find the largest value of K such that 10! is divisible by 30^K.

1. 1 2. 2 ⌘ 3. 3
4. 4 5. 5

10. Every student at Petrova School has at least one pet. The pets are all dogs, cats, or snakes. Some students own more than one type of pet, but no student who owns a dog also owns a snake.. There are 300 dog owners, 400 cat owners, and 50 snake owners among the students. A survey shows that there are 100 students who own both dogs and cats, while 20 own both cats and snakes. How many students attend Petrova School?

1. 630 ⌘ 2. 650 3. 680
4. 700 5. 710

11. For non-zero real numbers a and b, the line $2x+ay+1 = 0$ is parallel to the line $ax+by+2=0$ and is perpendicular to the line $bx-y-1=0$. Then a value of ab is

1. 2 2. -1 3. ½ ⌘
4. 4/3 5. none of these

12. What is the correct relationship between the numbers $\sqrt{2}$, $\sqrt[3]{3}$, $\sqrt[6]{6}$?

1. $\sqrt{2}<\sqrt[3]{3}<\sqrt[6]{6}$ 2. $\sqrt[3]{3}<\sqrt[6]{6}<\sqrt{2}$ 3. $\sqrt[6]{6}<\sqrt[3]{3}<\sqrt{2}$
4. $\sqrt[3]{3}<\sqrt{2}<\sqrt[6]{6}$ 5. none of these ⌘

13. What is the minimum number of data points that must be added to the list {-1,-1,-1,0,1} to decrease the value of the median?

1. 1 2. 2 3. 3
4. 4 5. 5 ⌘

14. If S is the set of all solutions (a,b) to the following system of inequalities, what is the maximum value of b?

$y \leq -5x + 200 \quad y \leq 20x$

1. 8
2. 160 ⌘
3. 200
4. 120
5. 220

15. What is the sum of the distinct prime divisors of 2024?

1. 23
2. 24
3. 28
4. 29
5. 36 ⌘

16. Aubrey, Bilin, Charles, David, and Eduardo are seated in a line and each seating order is equally likely. Which of the following is closest to the probability that there is exactly one person between Aubrey and Bilin?

1. 10%
2. 20%
3. 30% ⌘
4. 40%
5. 50%

17. A point P is chosen randomly from a triangular region in the Cartesian plane with vertices at (0,0), (4,0), and (0,4). What is the probability that P is at least one unit away from both the x and y axes?

1. 1/ 4 ⌘
2. 3/8
3. 9/16
4. 3/4
5. 1/8

18. A rectangular floor consists of 40 square tiles arranged in a 4x10 pattern. If a straight line is drawn diagonally from one corner to another, how many square will it pass through?

1. 10
2. 11
3. 12 ⌘
4. 13
5. 14

19. The line PQ is tangent to a circle of radius 1 centered at A and a circle of radius 3 centered at B, with points of tangency P and Q, respectively. The segment PQ intersects AB at R, a point between the circles. If PQ has length 6, what is the length of AR?

1. 1
2. $4/\sqrt{2}$
3. $\frac{\sqrt{13}}{2}$
4. $\sqrt{6}$
5. $\sqrt{\frac{13}{2}}$ ⌘

20. Find the smallest positive integer that can be divided by each of the numbers from 1 to 10 without leaving a remainder.

1. 120
2. 2550
3. 4660
4. 10!
5. none of these ⌘

21. Farmer Foghorn has 40 chickens that, collectively, lay 180 eggs per week. He is considering expanding his flock. Assuming the rates stay the same, what is the least number of chickens he would need to get 1000 eggs per month (for this problem – a month is 4 weeks)?

1. 50
2. 52
3. 53
4. 55
5. 56 ⌘

22. Let $f(x)=\frac{5x+7}{2x+3}$ Which of the following is $f^{-1}(x)$?

1. $\frac{-3x+4}{2x-3}$ 2. $\frac{3x-4}{2x-3}$ 3. $\frac{3x+4}{-2x+3}$
4. $\frac{3x+4}{2x+3}$ 5. $\frac{3x-7}{-2x+5}$ ⌘

23. 2024 is the sum of 23 consecutive numbers. What is the largest of these numbers?

1. 23 2. 77 3. 88
4. 99 ⌘ 5. 100

24. Two positive numbers *a* and *b* are such that $a+b=ab=a^2-b^2$. What is the value of *b* ?

1. $\frac{-1+\sqrt{5}}{2}$ 2. $\frac{-1+\sqrt{6}}{2}$ 3. $\frac{3-\sqrt{5}}{2}$
4. $\frac{1+\sqrt{5}}{\sqrt{6}}$ 5. $\frac{1+\sqrt{5}}{2}$ ⌘

25. An isosceles triangle has base length 40 and height 48. A semicircle is inscribed in the triangle such that the diameter of the semicircle is on the base of the triangle and the two other sides of the triangle are both tangent to the semicircle. The radius is closest to which of the following?

1. 18.4 ⌘ 2. 26.0 3. 13.7
4. 22.9 5. 20.0