CALIFORNIA STATE UNIVERSITY, BAKERSFIELD
Lee Webb Math Field Day 2024
Individual Medley, Junior-Senior Level

Your answers to these questions should be on the side of the answer sheet that has answer spaces A, B, C, D, E (NOT 1, 2, 3, 4, 5). On the answer sheet you should write your name, school name, level (Junior-Senior), and Division (your proctor should have a list of which schools are in which divisions).

For each of the following questions, blacken the appropriate circle on the answer sheet. Each correct answer is worth four points. **One point is deducted for each incorrect answer.** An unanswered question is given zero points. Note that random guessing may adversely affect your score.

You have 50 minutes to complete the examination. If you finish early, review your answers. When the exam is over, give your answer sheet to the proctor.

All calculators, cell phones, music players, and other electronic devices should be put away in backpacks, purses, pockets, etc. Leaving early or otherwise disrupting other contestants may be cause for disqualification.

1. In Miss Edser's class there are 23 students. Thirteen of them are in the Math Club. Eighteen students are in the Frisbee Club. Five students are not in either club. How many are in both clubs?

A. 5 B. 7 C. 12

D. 13 E. 17

2. When expressed in reduced form, what is the sum of the numerator and denominator of $\frac{10!+8!}{10!-8!}$

A. 9 B. 20 C. 180

D. 33588480 E. none of these

3. Evaluate $\sum_{k=0}^{2024}(-1)^{k^2+k}$

A. 1 B. 2 C. 0

D. -1 E. -2

4. Solve the following system of equations:

$$\frac{3}{x}-\frac{2}{y}=4$$
$$\frac{-7}{x}+\frac{5}{y}=-7$$

A. $(1/6,1/9)$ B. $(1/7,1/4)$ C. $(1/5,1/10)$

D. $(1/4,1/9)$ E. $(1/6,1/7)$

5. For how many integer values of n is $|12n^2+11n-15|$ prime?

A. 0 B. 1 C. 2

D. 4 E. infinitely many

6. What is the correct relationship between the numbers $\sqrt{2}$, $\sqrt[3]{3}$, $\sqrt[6]{6}$?

A. $\sqrt{2}<\sqrt[3]{3}<\sqrt[6]{6}$ B. $\sqrt[3]{3}<\sqrt[6]{6}<\sqrt{2}$ C. $\sqrt[6]{6}<\sqrt[3]{3}<\sqrt{2}$

D. $\sqrt[3]{3}<\sqrt{2}<\sqrt[6]{6}$ E. none of these

7. Suppose a, b, c are distinct natural numbers and the sum of their reciprocals is 1. What is the sum of a, b, c ?

A. *8* B. 10 C. *11*

D. 12 E. 14

8. Suppose *F(x)* is a function that satisfies *F(xy)=F(x)+F(y) -1* for all real numbers *x* and *y*. Also assume that *F(2) = 3*. Determine *F(64)*.

A. 4 B. 7 C. 10

D. 11 E. 13

9. Let *a, b, c,* and d be integers satisfying *alog 2 + blog3 + clog5 + dlog7 = 2024*. (the logarithm is base 10). What is the value of $\frac{a+b}{c+d}$?

A. 1 B. 3 C. 23/57

D. 17/2024 E. none of these

10. Aubrey, Bilin, Charles, David, and Eduardo are seated in a line and each seating order is equally likely. Which of the following is closest to the probability that there is exactly one person between Aubrey and Bilin?

A. 10% B. 20% C. 30%

D. 40% E. 50?

11. Suppose $x+\frac{1}{x}=5$. Determine $x^3+\frac{1}{x^3}$.

A. 1 B. 2 C. $\sqrt{2}$

D. $2\sqrt{2}$ E. $2\sqrt{3}$

12. The line PQ is tangent to a circle of radius 1 centered at A and a circle of radius 3 centered at B, with points of tangency P and Q, respectively. The segment PQ intersects AB at R. If PQ has length 6, what is the length of AR?

A. 1 B. $4/\sqrt{2}$ C. $\frac{\sqrt{13}}{2}$

D. $\sqrt{6}$ E. $\sqrt{\frac{13}{2}}$

13. A book starts with page number 1, and a total of 3293 digits were used to number the pages. What is the page number of the last page?

A. 1100 B. 1011 C. 1010

D. 1001 E. 999

14. If $x^2+y^2=3$ and $x^4+y^4=6$, find the value of $x^{12}+y^{12}$.

A. 216 B. 432 C. 345/2

D. 439/2 E. 431/2

15. Suppose the numbers 1,2,3,4,5,6 are re-ordered randomly and the labeled a, b, c, d, e, f. What is the probability that abc+def is an even number?

A. 1/6 B. 5/6 C. 3/4

D. 9/10 E. over 99%

16. If S is the set of all solutions (a,b) to the following system of inequalities, what is the maximum value of b?

$$y \leq -5x + 200 \qquad y \leq 20x$$

A. 8 B. 160 C. 200

D. 120 E. 220

17. Assume Mixture A is made of 20% water and Mixture B is made of 50% water. A barrel is partially filled with 40 liters of Mixture B. How many liters of Mixture A should be added to make a new mixture with 30% water (the barrel is large enough that you do not need to worry about it overflowing)?

A. 40 B. 50 C. 55.25

D. 80 E. 75

18. Find the smallest positive integer that can be divided by each of the numbers from 1 to 10 without leaving a remainder.

A. 3870 B. 2550 C. 5280

D. 10! E. none of these

19. How many distinct values in $0<\theta<\pi$ satisfy $2\cos(\theta)=7\sin(2\theta)$

A. 0 B. 1 C. 2

D. 4 E. infinitely many

20. The minute and hour hands of a clock have lengths 3 and $\sqrt{2}$, respectively. What is the distance between the tips of the hands at 4:30?

A. 2 B. $\frac{\pi}{2}$ C. $\sqrt{5}$

D. 3 E. none of these

21. ABCD is a rectangle with side AD having length 1. P is on edge AB such that DP and DB trisect angle ADC. What is the perimeter of triangle PBD?

A. $\frac{1+\sqrt{5}}{2}$ B. $3+\frac{\sqrt{3}}{3}$ C. $2+\sqrt{3}$

D. $2+\frac{4\sqrt{3}}{3}$ E. $2+\frac{3\sqrt{3}}{4}$

22. ABCDE is a pentagon with right angles at B, C, and E. The lengths of BC, CD, DE, EA, are 24, 27, 20, 15, respectively. What is the area of the pentagon?

A. 798 B. 714 C. 688

D. 747 E. 474

23. Determine of the coefficient of x^{17} in the expansion of $(1+x^5+x^7)^{20}$

A. 3400 B. 3420 C. 4320

D. 4400 E. 0

24. Murray wants to treat his four friends to ice cream cones. He has 8 flavors available. He asks each friend which flavor they want. Each friend only picks one flavor. How many different sets of "orders" can he get?

A. 7900 B. 7920 C. 8020

D. 8080 E. 9024

25. Two positive numbers *a* and *b* are such that $a+b=ab=a^2-b^2$. What is the value of a ?

A. $\frac{-1+\sqrt{5}}{2}$ B. $\frac{-1+\sqrt{6}}{2}$ C. $\frac{3+\sqrt{5}}{2}$

D. $\frac{1+\sqrt{5}}{\sqrt{6}}$ E. $\frac{1+\sqrt{5}}{2}$