CALIFORNIA STATE UNIVERSITY, BAKERSFIELD

Lee Webb Math Field Day 2023

Team Medley, Junior-Senior Level

Each correct answer is worth ten points. Answers require justification. Partial credit may be given. Unanswered questions are given zero points.

You have 50 minutes to complete the Exam. When the exam is over, give only one set of answers per team to the proctor. Multiple solutions to the same problem will invalidate each other.

Elegance of solutions may affect score and may be used to break ties.

All calculators, cell phones, music players, and other electronic devices should be put away in backpacks, purses, pockets, etc. Leaving early or otherwise disrupting other contestants may be cause for disqualification.

1. Prove that for $x \in (0, \pi/2)$, cos(cos(x)>sin(sin(x) and cos(cos(cos(cos(x)))) > sin(sin(sin(sin(x)))). You may assume that $\cos(x)+\sin(x) \leq \sqrt{2}$.

2. Assume that $a>0$, and $r>0$. Let $f(x)=ax^r$. Find a and r, given that $f'(x)=f^{-1}(x)$

3. How many subsets of {1,2,3,4,5,6,7,8,9,10,11,12} are there that contain at least one even number, and at least one multiple of 3, and at least one prime number?

4. Evaluate the following limit:

$$\lim_{N \to \infty} N^7 \prod_{k=1}^{N} \frac{k}{7+k}$$