CALIFORNIA STATE UNIVERSITY, BAKERSFIELD
Lee Webb Math Field Day 2023
Individual Medley, Junior-Senior Level

Your answers to these questions should be on the side of the answer sheet that has answer spaces A, B, C, D, E (NOT 1, 2, 3, 4, 5). On the answer sheet you should write your name, school name, level (Junior-Senior), and Division (your proctor should have a list of which schools are in which divisions).

For each of the following questions, blacken the appropriate circle on the answer sheet. Each correct answer is worth four points. **One point is deducted for each incorrect answer.** An unanswered question is given zero points. Note that random guessing may adversely affect your score.

You have 50 minutes to complete the examination. If you finish early, review your answers. When the exam is over, give your answer sheet to the proctor.

All calculators, cell phones, music players, and other electronic devices should be put away in backpacks, purses, pockets, etc. Leaving early or otherwise disrupting other contestants may be cause for disqualification.

1. In Miss Meanor's class there are 35 students. Twenty of them are in the Math Club. Fifteen students are in the Rocket Club. Five students are not in either club. How many are in both clubs?

A. 5 B. 7 C. 12
D. 16 E. 17

2. How many distinct real solutions does the equation $x^4-x^3+x^2-x+1=0$ have?

A. 0 B. 1 C. 2
D. 3 E. 4

3. Evaluate $\sum_{k=13}^{15}(-1)^{k^2+k}k$

A. 14 B. 21 C. 28
D. 35 E. 42

4. Solve the following system of equations:

$$\frac{4}{x}-\frac{2}{y}=4$$
$$\frac{2}{x}+\frac{1}{y}=18$$

A. $(1/6,1/10)$ B. $(1/8,1/4)$ C. $(1/3,1/10)$
D. $(1/4,1/9)$ E. $(1/5,1/8)$

5. A point P is randomly selected from a triangular region bounded by (0,0), (10,0), and (0,5). What is the probability that the point is within one unit from at least one of the axes?

A. $51/100$ B. $51/200$ C. $47/100$
D. $47/200$ E. $1/2$

6. Let a and r be positive real numbers. For the following set of numbers, what is the inter-quartile range?

$$a-2r,\ a-r,\ a,\ a+r,\ a+2r,\ a+3r,\ a+4r,\ a+5r,\ a+6r,\ a+7r$$

A. r B. $3.5r$ C. $a+4r$

D. $4.5r$ E. $5r$

7. Which of the following best describe the graph of the equation $x^{3y}-y^{3x}=0$?

A. a closed loop curve with 2 bulges and 2 dimples

B. two circles that intersect at the origin

C. four lines that intersect at the origin

D. eight lines – some intersect at the origin

E. two pairs of parallel lines

8. Points A and B are the ends of a diameter of circle S. Point C is another point on circle S. What is the minimum possible value of the measure of angle ACB (in degrees)?

A. 0 B. 30 C. 45

D. 60 E. 90

9. In the complex plane, how units apart are the solutions to x^2+x+1 ?

A. $3i$ B. $\sqrt{3}$ C. $\sqrt{3}/2$

D. $\sqrt{3}/3$ E. $\sqrt{3}/2i$

10. What is the difference between the largest and smallest possible values of $f(x)=\sin(x)+\cos(x)$?

A. 1 B. 2 C. $\sqrt{2}$

D. $2\sqrt{2}$ E. $2\sqrt{3}$

11. Tommy is trying to determine the PIN for a lock. He notices that on the 10 digit keypad, the buttons with 3, 4, and 7 are worn down considerably compared with the other keys. Assuming the correct PIN is a sequence of 5 digits that only use these 3 digits, how many possible sequences should Tommy consider trying?

A. 120 B. 240 C. 360
D. 480 E. 600

12. A dress has 200 decorative beads. 70% of them are green and the rest are blue. The costume manager sews additional beads onto the dress in the following pattern – blue, blue, green. How many beads are on the dress when the percentage of blue beads first reaches (or surpasses) 50%?

A. 435 B. 436 C. 437
D. 439 E. 440

13. Thirty-two table tennis players enter a triple-elimination tournament. This means that a player is not eliminated until they have lost 3 games. In the first round, everyone is paired up randomly. In the second round, winners from the first round are paired with each other and losers are paired with each other. After that, players are paired with other players who have the same number of wins and losses as they do, or as close as possible to this. Assuming that there are never any ties (every game has a winner and a loser), how many games must be played before there is an overall winner of the tournament?

A. 62 B. 64 C. 87
D. 92 E. 93

14. The total number of integers that are between 1 and 101 that are multiples of 4 or 6 or both is:

A. 37 B. 41 C. 33

D. 29 E. 25

15. If a standard die is rolled 3 times, what is the probability that the first value is greater than the sum of the other two values?

A. 5/108 B. 5/54 C. 7/24

D. 2/27 *E*. 17/216

16. Point C is the center of two circles . A and D are on the outer circle and chord AD is tangent to the inner circle at B. If AD=24 and BC = 5, what is the area of the annulus between the two circles?

A. 25π B. 81π C. 100π

D. 125π E. 144π

17. Three sides of a triangle all have integer lengths and those lengths are in arithmetic progression. The shortest side has length that is 23% of the perimeter. What is the length of the longest side of the smallest possible triangle that meets these conditions.

A. 131 B. 144 C. 165

D. 192 E. No such triangle exists.

18. ABCD is a square with side length 1. AXY is an equilateral triangle such that B is outside the triangle, while C is on XY and D is on AY. What is the area of AXY?

A. $\frac{\sqrt{3}+3}{2}$ B. $\frac{\sqrt{3}+3}{3}$ C. $\frac{\sqrt{3}}{3}+\frac{3}{4}$

D. $\frac{\sqrt{3}}{3}+\frac{1}{2}$ E. $\frac{\sqrt{3}}{3}+1$

19. Jim calculated that the solution of a question was equal to 99!/94! He multiplied this out to be 8582d77280, where the d represents a digit he couldn't read when he went back over his notes. What is the correct value of d?

A. 2 B. 3 C. 7

D. 8 E. 0

20. Joe has 30 cards. For each number 1, 2, 3, …, 10 and for each color red, blue, green, there is a card. From Joe's deck of cards, how many pairs can be made such that the values on the cards do not match?

A. 400 B. 405 C. 800

D. 805 E. 810

21. Your ship sends messages by hoisting flags on a line (rope). You have four distinct flags. Assuming a message must have at least one flag, how many different messages can you send?

A. 24 B. 48 C. 52

D. 60 E. 64

22. Simplify $\sin(5\pi/12)\cos(\pi/12)$

A. $\frac{\sqrt{2}+\sqrt{6}}{4}$ B. $\frac{\sqrt{6}-\sqrt{2}}{4}$ C. $\frac{\sqrt{3}+2}{8}$

D. $2-\sqrt{3}$ E. $\frac{2+\sqrt{3}}{4}$

23. Simplify $\frac{5\log_5(125)}{\frac{1}{2}\log_3(243)}$

A. 1/6 B. 1/4 C. 1

D. 23/6 E. $\log_{20} 6$

24. Suppose $b^{\log_5 11}=8$. Then, solving for *b* gives $b=2^{(\ln 5)^r}$ for what value of *r*?

A. $\frac{3}{\ln 11}$ B. $\frac{8}{\ln 11}$ C. $\frac{11}{\ln 8}$

D. $\frac{-3}{\ln 8-\ln 11}$ E. $\frac{11}{\ln 3}$

25. There are 12 steps to get to the main door of Trigos Elementary School. Javier can take 1 or 2 steps at a time. In how many ways can Javier climb the steps?

A. 200 B. 213 C. 236

D. 276 E. 302