

# 49th Lee Webb Math Field Day

California State University, Bakersfield  
Department of Mathematics

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# Varsity Math Bowl

## Varsity Math Bowl Round 1 Sample Question

2020 is not a prime. If you double this number and then add 1, you get a number that's still not prime. But you do get something that's 4000 more than a prime. What prime is that?

## Varsity Math Bowl Round 1 Question 1

Suppose  $\frac{1}{5}$  of  $5^{10}$  equals  $5^j$ . What is the value of  $j$ ?

## Varsity Math Bowl Round 1 Question 2

Suppose  $f(x) = 2x - 1$ .

Evaluate  $f(f(f(f(f(f(1))))))$ .

## Varsity Math Bowl Round 1 Question 3

Suppose the two legs of a right triangle have lengths  $10\sqrt{10}$  and  $\sqrt{10}\sqrt{101}$  and the length of the hypotenuse is  $\sqrt{n}$ . What is the value of  $n$ ?

## Varsity Math Bowl Round 1 Question 4

What is one forty-ninth of seven cubed?

## Varsity Math Bowl Round 1 Question 5

How many three letter strings are there that are all vowels and don't have repeated letters (here 'y' does not count as a vowel)?



## Varsity Math Bowl Round 1 Question 6

Let  $S$  be the set of the first 100 positive numbers that are 1 less than a multiple of 7. What is the average of the numbers in  $S$ ?

## Varsity Math Bowl Round 1 Question 7

An isosceles triangle has two sides of length 1 and the angle in between them is  $30^\circ$ . What is the area of the triangle?

## Varsity Math Bowl Round 1 Question 8

Segments  $AB$  and  $CD$  are parallel and have lengths 10 and 6 respectively.  $BC$  and  $AD$  intersect at  $P$ .  $BC$  has length 30. What is the length of  $BP$ ?

## Varsity Math Bowl Round 1 Question 9

Simplify:

$$\frac{8!6!4!}{3!5!7!}$$

## Varsity Math Bowl Round 1 Question 10

What are the first 4 digits of  $200^{10}$ , after expanding?

Round 2

## Varsity Math Bowl Round 2 Sample Question

Simplify

$$\frac{2020}{20}.$$

## Varsity Math Bowl Round 2 Question 1

What is the value of

$$4 + ((6 + 7) + (3 + 8)) + 2$$



## Varsity Math Bowl Round 2 Question 2

What is the product of all the solutions to

$$\frac{20}{t} = \frac{200}{t^2}?$$

## Varsity Math Bowl Round 2 Question 3

If all the numbers in the following list are increased by 100%, then the average will increase by a factor of what number?

44, 24, 46, 56, 67, 37, 77, 18, 38

## Varsity Math Bowl Round 2 Question 4

In triangle  $ABC$ ,  $\sec A = \sqrt{10}$ . What is  $\tan A$ ?

## Varsity Math Bowl Round 2 Question 5

What is the average of the numbers in the set

$$\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}\right\}?$$

## Varsity Math Bowl Round 2 Question 6

A regular six pointed star is formed by placing one equilateral triangle over another. What percent of the area of the star is common to both triangles?

## Varsity Math Bowl Round 2 Question 7

Solve for  $n$ :

$$n^{n+1} - (n + 1)^n = 7849$$

## Varsity Math Bowl Round 2 Question 8

Solve:

$$\log_{3x} 512 = 9$$

## Varsity Math Bowl Round 2 Question 9

Let  $a, b, c, d, e = 1, 2, 3, 4, 5$ , respectively.  
Simplify:

$$\left( \frac{((a - b + c - d + e)!)^4}{((-a + b - c + d - e)^2)!} \right)^{-1}$$



## Varsity Math Bowl Round 2 Question 10

Simplify

$$\log_3 \left( \frac{9^{4^3}}{3^{7^2}} \right)$$

Round 3

## Varsity Math Bowl Round 3 Sample Question

Evaluate

$$\frac{8000/200}{2 + 0 + 2 + 0 + 2 + 0 + 2}$$

## Varsity Math Bowl Round 3 Question 1

Which positive integer  $n$  satisfies:

$$1000 < n^{2^n} < 1000000 - 1$$

## Varsity Math Bowl Round 3 Question 2

Let

$$f(x) = \frac{x^4}{5} + \frac{x^2 - 40}{3}.$$

What is  $f(10)$ ?

## Varsity Math Bowl Round 3 Question 3

What is the greatest of the solutions to the equation:

$$\frac{1}{1-x} + \frac{2}{2-x} = 2$$

## Varsity Math Bowl Round 3 Question 4

Solve for  $x$ :

$$\frac{5^{12x}}{5^{12x^2}} = 125$$

## Varsity Math Bowl Round 3 Question 5

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## Varsity Math Bowl Round 3 Question 6

Simplify:

$$\left\lceil 20 / \left( \frac{2020}{202} \right) + 20.20 \right\rceil,$$

where  $\lceil x \rceil$  denotes the ceiling function.

## Varsity Math Bowl Round 3 Question 7

Evaluate:

$$\lim_{x \rightarrow \infty} \frac{3x^2 + 3/x}{x^4/3x^2 - 5/x^2}$$

## Varsity Math Bowl Round 3 Question 8

A car dealership has 9 sportscars. They are identical except that 3 are red, 3 are blue, and 3 are green. In how many different ways can they lined up on the front lawn to attract customers?

## Varsity Math Bowl Round 3 Question 9

For positive integers  $a, b$ , we have

$$\int_2^4 \frac{x+1}{x^2+2x+8} dx = \frac{1}{a} \ln b.$$

Determine  $a + b$ .

## Varsity Math Bowl Round 3 Question 10

A third degree polynomial has the form

$$x^3 + ax^2 + 63x + c.$$

Assuming two of the roots are 3 and 5,  
what is the third root?

Round 4

## Varsity Math Bowl Round 4 Sample Question

A sailboat can cross the ocean in 18 days. If, in addition, on the third day, another sailboat sets sail from the opposite shore, how long will it take the first sailboat to cross?

## Varsity Math Bowl Round 4 Question 1

Two positive integers, 24 and  $n$  are such that their greatest common divisor is 4 and their least common multiple is 192. What is  $n$ ?



## Varsity Math Bowl Round 4 Question 2

What is the harmonic mean of 4 and 6?

## Varsity Math Bowl Round 4 Question 3

Simplify

$$7^3 + 3 \cdot 7^2 \cdot 3 + 3 \cdot 7 \cdot 3^2 + 3^3$$

## Varsity Math Bowl Round 4 Question 4

Evaluate

$$\int_e^1 \ln x \, dx$$

## Varsity Math Bowl Round 4 Question 5

Three fair, standard, six-sided dice are rolled. What is the probability that the dice will show the three even numbers?

## Varsity Math Bowl Round 4 Question 6

Let

$$f(x, y) = \frac{x^4 - y^4}{x^2 - y^2}.$$

What is

$$\sqrt{f(3, 4)}?$$

## Varsity Math Bowl Round 4 Question 7

A curve is defined by the equation  $\log_x y = 2$ . What is the slope of the tangent line when  $x = 3$ ?

## Varsity Math Bowl Round 4 Question 8

Evaluate

$$\int_{-5}^5 x \int_1^3 x \, dx \, dx$$

## Varsity Math Bowl Round 4 Question 9

In a standard completed Sudoku game, what is the sum of all the entries in the grid?



## Varsity Math Bowl Round 4 Question 10

Evaluate

$$\log_{\int_4^9 dx} (30^2 + 40^2) - \log_{\tan(\tan^{-1} 5)} 4$$

## The End

Please be patient while we calculate the scores.

Closing Ceremony to commence shortly