CALIFORNIA STATE UNIVERSITY, BAKERSFIELD
MATHEMATICS FIELD DAY 2022

Team Medley, Varsity Level

There are 6 problems. You will have 50 minutes. You should only write your solution and relevant work legibly on this paper. Use separate scratch paper for work that may not contribute meaningfully to your solution. When the time is up, you should convert and send your work as one pdf file. Only pdf files will be accepted. Each correct answer is worth ten points. Partial credit may be given only for substantial progess towards solution.

Calculators, cellphones, and other electronic devices are not allowed.

GOOD LUCK!

1. In triangle ABC where $AB = 4$, $BC = 5$, and $CA = 8$. Furthermore, there is a point D on AC such that $CD = 3$ and $AD = 5$. What is the length of the segment BD?

2. Find all positive integers n such that $n^{\frac{1}{n-7}}$ is an integer.

3. Consider a square whose side is of length of 2 inches. If there are 5 points inside the square, prove that there exist at least one pair of the points such that the distance between the two points is less than $\sqrt{2}$.

4. Evaluate the following sum

$$\sum_{k=1}^{n} \frac{1}{k!\,(k+2)} = \frac{1}{1!\,3} + \frac{1}{2!\,4} + \frac{1}{3!\,5} + \cdots + \frac{1}{n!\,(n+2)}$$

5. Let C_k denote the circle whose equation is $x^2 + y^2 + 2kx - 2ky - 2k - 4 = 0$, where k is a real number. Are there any points through which the circle C_k passes for every k? If so, find them. If not prove it.

6. Thirty two table tennis players, all exactly the same level of ability enter a single-elimination tournament. The pairings are arranged randomly. What is the probability that two specific players, who are friends, will have to play each other?