CALIFORNIA STATE UNIVERSITY, BAKERSFIELD
MATHEMATICS FIELD DAY 2022

Individual Medley, Varsity Level

There are 25 problems. You will have 50 minutes. You should mark your answers clearly on this paper. When the time is up, you should transfer your answers into the google form with the link provided by your teacher. Each correct answer is worth four points. Each incorrect answer will receive a one point deduction. Answers left blank will be counted as zero points.

Calculators, cellphones, and other electronic devices are not allowed.

GOOD LUCK!

1. If $-1 < x < 3$, then $|x-3| + |x+1|$ equals

   (A) $2x - 2$

   (B) 4

   (C) $4 - x$

   (D) $2 - 2x$

   (E) None of the above

2. Suppose $f(n)$ is a function such that $f(1) = 1$, $f(2) = 2$, and $f(n+2) = f(n) + 2f(n+1)$ for all natural numbers n. Then $f(5)$ equals

   (A) 5

   (B) 12

   (C) 15

   (D) 23

   (E) None of the above

3. If $\frac{x-3y}{x+y} = 7$, what is the value of $\frac{y}{x}$?

   (A) 1

   (B) $\frac{3}{4}$

   (C) $-\frac{2}{3}$

   (D) $-\frac{3}{5}$

   (E) It cannot be determined.

4. The units digit of 1357^{39} is

   (A) 3

   (B) 7

   (C) 9

   (D) 1

   (E) None of the above

5. The equation $2x + 3y + 4z = 36$ describes a plane in 3-dimensional space. This plane intersects the x, y, and z axes at specific values. What is the sum of these values?

(A) 37

(B) 39

(C) 41

(D) 45

(E) None of the above

6. What is the units digit of $\sum_{n=1}^{20} n!$?

(A) 1

(B) 3

(C) 5

(D) 7

(E) None of the above

7. A point P is randomly selected from a triangular region bounded by $(0, 0), (4, 0)$ and $(0, 4)$. What is the probability that P is at least one unit away from both of the axes?

(A) $\frac{3}{4}$

(B) $\frac{1}{2}$

(C) $\frac{1}{4}$

(D) $\frac{1}{8}$

(E) None of the above

8. The sum of two positive numbers is 12. The reciprocal of one number is one fifth of the other number. What is the sum of the squares of the two numbers?

(A) 60

(B) 100

(C) 125

(D) 134

(E) None of the above

9. Solve the equation $e^{4x} + 2e^{2x} = 15$.

(A) $\ln \frac{5}{3}$

(B) $\frac{\ln 3}{\ln 2}$

(C) $\ln \sqrt{3}$

(D) $\ln \sqrt{5}$

(E) None of the above

10. Let $n = 77777 \times 99999$. What is the sum of the digits of n?

(A) 36

(B) 39

(C) 45

(D) 54

(E) None of the above

11. In how many different ways can 8 people be divided into two groups, one with 3 people and the other with 5 people?

(A) 15

(B) 56

(C) 336

(D) 40320

(E) None of the above

12. Let $f(x) = ax^3 + cx + d$ and $g(x) = \frac{x}{\sqrt[3]{a}}$. Suppose that $f\big(g(0)\big) = 4$ and $f\big(g(1)\big) = 7$. What is the value of $\frac{c^3}{a}$?

(A) 3

(B) 28

(C) $\frac{27}{2}$

(D) $\frac{\sqrt[3]{4}}{6}$

(E) None of the above

13. Let $f(x) = 2x^2 + 3x + 1$. For which value of k will the graph of $y = f(x - k)$ be symmetric with respect to the y-axis?

(A) $\frac{3}{2}$

(B) $-\frac{3}{2}$

(C) $\frac{3}{4}$

(D) $-\frac{3}{4}$

(E) None of the above

14. Let $z = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$, where $i^2 = -1$. What is z^{33}?

(A) $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$

(B) $-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$

(C) $-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i$

(D) i

(E) None of the above

15. For some natural numbers a and b, we have $\sqrt{69 + 28\sqrt{5}} = a + b\sqrt{5}$. What is $a + b$?

(A) 1

(B) 3

(C) 5

(D) 7

(E) None of the above

16. For points (x, y) on the circle $x^2 + y^2 = 1$, the maximum value of $x^2 + y$ is

(A) 0

(B) 1

(C) 2

(D) $\frac{4+\sqrt{2}}{2}$

(E) None of the above

17. A semicircle has radius 10. What is the area of an inscribed square?

(A) 40

(B) $40\sqrt{2}$

(C) $40\sqrt{10}$

(D) 80

(E) None of the above

18. Points A, B, C, and D are each on a different side of a square with side length 1 and form a rectangle. If $AB = \frac{1}{2}$, what is the area of $ABCD$?

(A) $\frac{2\sqrt{2}-1}{4}$

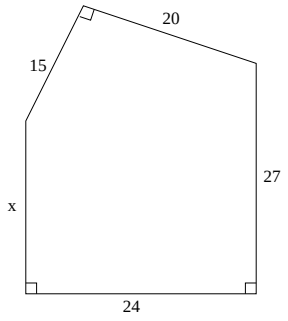
(B) $\frac{2\sqrt{2}-1}{2}$

(C) $\frac{4\sqrt{2}-1}{4}$

(D) $\frac{1}{4}$

(E) None of the above

19. What is the area of the pentagon shown here with sides of length 15, 20, 27, 24, and x inches?



(A) 798 in^2

(B) 714 in^2

(C) 688 in^2

(D) 648 in^2

(E) It cannot be determined without the value of x

20. $ABCD$ is a rectangle with $AB = 2$ and $BC = 3$. E is the midpoint of DA. Segments AC and BE divide the rectangle into 3 triangles and a quadrilateral. What is the area of the quadrilateral?

(A) 2

(B) $\frac{5}{2}$

(C) $2\sqrt{2}$

(D) 3

(E) None of the above

21. Given that $\cos(108^\circ) = \frac{1-\sqrt{5}}{4}$, what is $\sin(54^\circ)$?

(A) $\frac{1-\sqrt{5}}{4}$

(B) $\frac{3-\sqrt{5}}{4}$

(C) $\sqrt{\frac{3+\sqrt{5}}{8}}$

(D) $\sqrt{\frac{3-\sqrt{5}}{8}}$

(E) None of the above

22. A circle has radius 6. A chord of the circle is a perpendicular bisector of a radius. The chord divides the circle into two pieces. What is the area of the smaller pieces?

(A) $36\pi - 9\sqrt{3}$

(B) $12\pi - 9\sqrt{3}$

(C) $12\pi + 3\sqrt{3}$

(D) $6\pi - 3\sqrt{3}$

(E) None of the above

23. Suppose that $a^{\log_2 5} = 8$. What is $a^{(\log_2 5)^2}$?

(A) 125

(B) 343

(C) 625

(D) 729

(E) None of the above

24. Michael, Kayla, and Peter together ate 8 sugar cookies. Each of them ate at least one cookie. How many ways could this have happened?

(A) 6

(B) 10

(C) 21

(D) 56

(E) None of the above

25. Find the sum of all integer values of a for which the following circle has no x-intercept.

$$x^2 + y^2 + (a-2)x + 2ay + a - 2 = 0$$

(A) 0

(B) 3

(C) 6

(D) 12

(E) None of the above