CALIFORNIA STATE UNIVERSITY, BAKERSFIELD
MATHEMATICS FIELD DAY 2022

Individual Medley, Junior Varsity Level

There are 25 problems. You will have 50 minutes. You should mark your answers clearly on this paper. When the time is up, you should transfer your answers into the google form with the link provided by your teacher. Each correct answer is worth four points. Each incorrect answer will receive a one point deduction. Answers left blank will be counted as zero points.

Calculators, cellphones, and other electronic devices are not allowed.

GOOD LUCK!

1. The least common multiple of 45 and 60 is

   (A) 90

   (B) 180

   (C) 360

   (D) 2700

   (E) None of the above

2. If $-1 < x < 3$, then $|x-3| + |x+1|$ equals

   (A) $2x - 2$

   (B) 4

   (C) $4 - x$

   (D) $2 - 2x$

   (E) None of the above

3. Under which condition is $\dfrac{xy}{x-y}$ negative?

   (A) $0 < y < x$

   (B) $y < x < 0$

   (C) $x < 0 < y$

   (D) $x < y < 0$

   (E) None of the above

4. If $\dfrac{2x-3y}{2x+y} = 7$, what is the value of $\dfrac{y}{x}$?

   (A) 1

   (B) $\frac{2}{3}$

   (C) $-\frac{2}{3}$

   (D) $-\frac{6}{5}$

   (E) It cannot be determined.

5. Suppose $f(n)$ is a function such that $f(1) = 1$, $f(2) = 2$, and $f(n+2) = f(n) + 2f(n+1)$ for all natural numbers n. Then $f(5)$ equals

(A) 5

(B) 12

(C) 15

(D) 23

(E) None of the above

6. If $x > 0$ and $\left[\dfrac{(x^n)^3 x^2}{x^n}\right]^{\frac{1}{2}} = \dfrac{1}{\sqrt{x}}$, then n equals

(A) 1

(B) $\frac{1}{2}$

(C) $-\frac{1}{2}$

(D) $-\frac{3}{2}$

(E) None of the above

7. For nonzero real numbers a and b, the line $2x + ay + 1 = 0$ is parallel to the line $ax + by + 2 = 0$ and is perpendicular to the line $bx - y - 1 = 0$. Then, the value of ab is

(A) 2

(B) -1

(C) $\frac{1}{2}$

(D) $\frac{4}{3}$

(E) None of the above

8. The unit digit of 1357^{39} is

(A) 1

(B) 3

(C) 7

(D) 9

(E) None of the above

9. Julie drove 390 miles in 8 hours. First, she went for a while at 40 mi/hr and then drove the rest of the way at 60 mi/hr. How many miles did she drive at the slower speed?

(A) 180

(B) 200

(C) 220

(D) 240

(E) None of the above

10. A line with positive slope intersects the y-axis at an angle of $30°$. What is the slope of the line?

(A) $\frac{\sqrt{3}}{3}$

(B) $\frac{\sqrt{3}}{2}$

(C) 2

(D) $\sqrt{3}$

(E) None of the above

11. At most how many regions do four lines can divide the plane into?

(A) 8

(B) 10

(C) 12

(D) 16

(E) None of the above

12. Chef Toullie wants to put 6 dishes in a line on the buffet table. How many ways can he do this if the 3 meat dishes have to be together and the 3 vegetarian dishes have to be together?

(A) 12

(B) 72

(C) 120

(D) 720

(E) None of the above

13. Let $n = 77777 \times 99999$. What is the sum of the digits of n?

(A) 36

(B) 39

(C) 45

(D) 54

(E) None of the above

14. If $x^2 - 1$ is a factor of $2x^3 + ax^2 + bx - 1$, then $a - 2b$ equals

(A) -1

(B) 0

(C) 1

(D) 3

(E) None of the above

15. In how many different ways can 8 people be divided into two groups, one with 3 people and the other with 5 people?

(A) 15

(B) 56

(C) 336

(D) 40320

(E) None of the above

16. Three lines all go through the point $(3, 3)$. The slopes of the lines are 1, 2, and 3. What is the absolute difference between the highest and lowest of their y-intercepts?

(A) 2

(B) 4

(C) 6

(D) 9

(E) None of the above

17. Suppose that $nx + 35 = 3n$. For how many integer values of n is x also an integer?

(A) 0

(B) 4

(C) 6

(D) 8

(E) None of the above

18. It takes 8 hours for Julio to clean a house and it takes 6 hours for Julio and Kate to clean the same house together. How long will it take for Kate to clean the house alone?

(A) 6 hours

(B) 12 hours

(C) 24 hours

(D) 36 hours

(E) None of the above

19. A point P is randomly selected from a triangular region bounded by $(0, 0), (4, 0)$ and $(0, 4)$. What is the probability that P is at least one unit away from both of the axes?

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) $\frac{3}{4}$

(D) $\frac{1}{8}$

(E) None of the above

20. Triangle A has vertices at $(-1, 0), (3, 0)$, and $(1, 4)$. Triangle B is obtained by reflecting A around the y-axis. What is the enclosed area of the union of A and B?

(A) 14

(B) 15

(C) 16

(D) 18

(E) None of the above

21. What is the tens digit of 7^{2022}?

(A) 1

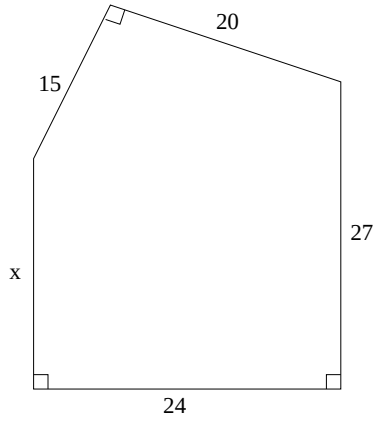
(B) 4

(C) 7

(D) 9

(E) None of the above

22. What is the area of the pentagon shown here with sides of length 15, 20, 27, 24, and x inches?



(A) 798 in^2

(B) 714 in^2

(C) 688 in^2

(D) 648 in^2

(E) It cannot be determined without the value of x.

23. Square $ABCD$ has side length 4. Points E and F are on AB and BC respectively, and both E and F are 1 unit away from B. Diagonal AC intersects DE and DF at G and H. What is the area of the pentagon $EBFHG$?

(A) $\frac{18}{7}$

(B) $\frac{20}{7}$

(C) $\frac{24}{13}$

(D) $\frac{25}{3}$

(E) None of the above

24. Michael, Kayla, and Peter together ate 8 sugar cookies. Each of them ate at least one cookie. How many ways could this have happened?

(A) 6

(B) 10

(C) 21

(D) 56

(E) None of the above

25. Find the sum of all integer values of a for which the following circle has no x-intercept.

$$x^2 + y^2 + (a-2)x + 2ay + a - 2 = 0$$

(A) 0

(B) 3

(C) 6

(D) 12

(E) None of the above