CALIFORNIA STATE UNIVERSITY, BAKERSFIELD

Lee Webb Math Field Day 2019

Team Medley, Junior-Senior Level

Each correct answer is worth ten points. Answers require justification. Partial credit may be given. Unanswered questions are given zero points.

You have 50 minutes to complete the Exam. When the exam is over, give only one set of answers per team to the proctor. Multiple solutions to the same problem will invalidate each other.

Elegance of solutions may affect score and may be used to break ties.

All calculators, cell phones, music players, and other electronic devices should be put away in backpacks, purses, pockets, etc. Leaving early or otherwise disrupting other contestants may be cause for disqualification.

1. Show that for any natural number *n*, greater than 1, the quantity n^4+4 is not prime.

2. There are two urns. The first contains 5 white balls. The second contains 4 white balls and one black ball. An urn is selected randomly and then a ball from it is selected randomly and removed. This is repeated until one of the urns is empty. What is the probability that the black ball is still in its urn?

3. On a 5x5 "chess board", assume a rook (a piece that can move horizontally or vertically as many spaces as desired) is trying to get from the lower left square to the upper right square. Further, assume that on each move it only goes up or only goes right (no backtracking). How many sequences of moves are there for the rook to do this? Note that a "sequence of moves" is different from a "path". For example, to get from the lower left to the square that is two to the right and two up, there are 6 paths, but 14 sequences of rook moves.

4. In triangle ABC, BC has length 14, and the other two sides have length 25. C1 is the inscribed circle. C2 is inscribed so that it is tangent to C1 and to sides AB and AC. C3 is inscribed so that it is tangent to C2 and to sides AB and AC. These definitions continue for an infinite sequence of circles. What is the sum of the circumferences and the areas of all the circles?

5. How many ways can the numbers 1, 2, 3, 4, 5, 6, 7 be arranged so that no number is in its original position?

6. What is the smallest possible area of a quadrilateral ABCD that meets the following conditions: diagonals AC and BD meet at P, inside ABCD and triangles ABP and CDP have areas 4 and 9, respectively?