CALIFORNIA STATE UNIVERSITY, BAKERSFIELD

Lee Webb Math Field Day 2019

Team Medley, Freshman-Sophomore Level

Each correct answer is worth ten points. Answers require justification. Partial credit may be given. Unanswered questions are given zero points.

You have 50 minutes to complete the Exam. When the exam is over, give only one set of answers per team to the proctor. Multiple solutions to the same problem will invalidate each other.

Elegance of solutions may affect score and may be used to break ties.

All calculators, cell phones, music players, and other electronic devices should be put away in backpacks, purses, pockets, etc. Leaving early or otherwise disrupting other contestants may be cause for disqualification.

1. How many four digit numbers are there such that the first and third digits have the same sum as the second and fourth digits?

2. Show that for any natural number *n*, greater than 1, the quantity n^4+4 is not prime.

3. Show that for any numbers, *a, b, c,* it is true that $a^2+b^2+c^2 \geq ab+bc+ca$

4. In triangle ABC, suppose we have lengths AB=24, BC=28, CA = 32. Given any point P inside the triangle, we can construct rays AP, BP, and CP that intersect the sides of the triangle at X, Y, and Z. Thus triangle ABC is partitioned into 6 smaller triangles: AZP, ZBP, BXP, XCP, CYP, YAP. For how many points P, do all 6 of these triangles have the same area?

5. How many ways can the numbers 1, 2, 3, 4, 5, 6 be arranged so that no number is in its original position?

6. What is the smallest possible area of a quadrilateral ABCD that meets the following conditions: diagonals AC and BD meet at P, inside ABCD and triangles ABP and CDP have areas 4 and 9, respectively?