CALIFORNIA STATE UNIVERSITY, BAKERSFIELD
Lee Webb Math Field Day 2019
Individual Medley, Junior-Senior Level

Your answers to these questions should be on the side of the answer sheet that has answer spaces A, B, C, D, E (NOT 1, 2, 3, 4, 5). On the answer sheet you should write your name, school name, level (Junior-Senior), and Division (your proctor should have a list of which schools are in which divisions).

For each of the following questions, blacken the appropriate circle on the answer sheet. Each correct answer is worth four points. **One point is deducted for each incorrect answer.** An unanswered question is given zero points. Note that random guessing may adversely affect your score.

You have 50 minutes to complete the examination. If you finish early, review your answers. When the exam is over, give your answer sheet to the proctor.

All calculators, cell phones, music players, and other electronic devices should be put away in backpacks, purses, pockets, etc. Leaving early or otherwise disrupting other contestants may be cause for disqualification.

1. In Miss Meanor's class there are 32 students. Twenty of them are in the Math Club. Fifteen students are in the Geology Club. Five students are not in either club. How many are in both clubs?

A. 8 B. 12 C. 15
D. 16 E. 17

2. How many distinct solutions does the equation $3x^4+3x^2=6x^3$ have?

A. 1 B. 2 C. 3
D. 4 E. 5

3. Evaluate $\sum_{k=12}^{14}(k^2-k)$.

A. 470 B. 480 C. 492
D. 500 E. 509

4. What is the area of a triangle with vertices at (2,4), (4,3), and (7, 10)?

A. 8 B. 10 C. 12
D. 8.5 E. 10.5

5. Solve $e^{4x}-2e^{2x}=15$

A. $\ln 3-\ln 2$ B. $\frac{\ln 5}{\ln 2}$ C. $\ln\sqrt{5}$
D. $\ln 4$ E. $\ln 3$

6. The arrow on a spinner is pointing directly North. Each time it is flicked, it goes clockwise 64 degrees. How many flicks will it take before it is pointing directly North again?

A. 24 B. 30 C. 32
D. 36 E. 45

7. How many integer values of a are there such that $\ln 6$, $\ln 18$, and $\ln a$ could be the side lengths of a triangle?

A. 45 B. 57 C. 100

D. 104 E. ∞

8. The equation $3x+4y+5z=60$ describes a plane in 3-dimensional space. This plane intersects the x,y, and z axes at specific values. What is the sum of these values?

A. 43 B. 47 C. 51

D. 62 E. 65

9. What is the absolute value of the product of all the solutions to $|||x+1|-2|+3|=2019$?

A. 2019 B. 356823 C. 4072323

D. 2435654765 E. 8845457214

10. How far apart are the roots of $6x^2-7x-20$?

A. 4 B. 23/6 C. 16/3

D. 5 E. 6

11. Let n = 44444 x 99999. What is the sum of the digits of n?

A. 36 B. 39 C. 45

D. 54 E. 90

12. The horizontal line segment that has one endpoint on the parabola $y=ax^2$ and the other endpoint at the focus of the parabola has length 12. What is the value of a?

A. $1/5$ B. $1/6$ C. $1/12$
D. $1/24$ E. $\sqrt{3}/10$

13. For some natural numbers a, and b, we have $\sqrt{73+28\sqrt{6}}=a+b\sqrt{6}$. What is $a+b$?

A. 7 B. 9 C. 11
D. 13 E. 14

14. A soda machine dispenses three flavors of soda: Calculus Cola, Square-Root Beer, and Infizznity. But the different flavored bottles come out randomly. What is the minimum number of sodas that must be purchased so that the four Minusketeers can each have the same flavor?

A. 8 B. 9 C. 10
D. 11 E. 13

15. If a standard die is rolled 3 times, what is the probability that the first value is greater than both of the other two values?

A. 5/18 B. 55/216 C. 7/24
D. 61/216 *E*. 67/216

16. If all the numbers 1, 2, 3, …, 10000 are written out, how many times will the digit 7 appear?

A. 2358 B. 2944 C. 3600
D. 3439 E. 4000

17. Paul added up all the page numbers in a booklet that started with page 1 on the front page. He obtained the sum 888. He added correctly but did not notice that one page had fallen out. What is the sum of the page numbers on the front and back of the missing page?

A. 15 B. 23 C. 27
D. 31 E. 35

18. On segment AD is a point E and on segment BC is a point F such that ABCD is a square and such that segments BE, EF, and FD all have length 100. What is the area of ABCD?

A. 8000 B. 9000 C. 10000
D. $10000\sqrt{5}$ E. $6000\sqrt{15}$

19. On a table of factorial values, the entry for 23! had two digits that were illegible. It read $23!=2585201--38884976640000$. The missing two digits are

A. 49 B. 04 C. 76
D. 33 E. 67

20. A circle has radius 7. A chord of the circle is a perpendicular bisector of the radius. The chord divides the circle into two pieces. What is the area of the larger piece?

A. $\frac{392\pi+81\sqrt{3}}{12}$ B. $\frac{384\pi+81\sqrt{3}}{12}$ C. $\frac{392\pi+27\sqrt{3}}{12}$
D. $\frac{384\pi+181\sqrt{3}}{12}$ E. $\frac{392\pi+147\sqrt{3}}{12}$

21. In the country of Baloney 10% of the citizens are traitors. Colonel X, a counter-intelligence agent, can pick out a traitor during an interview, nine times out of 10. But, perhaps because he is a bit paranoid, when he interviews a loyal citizen, he still misidentifies them as traitors two times out of 10. When Colonel X interviews Citizen K, he declares him to be a traitor. What is the probability that Col. X is correct (estimate to two digit accuracy)?

A. 13% B. 18% C. 24%
D. 27% E. 30%

22. Along an inside edge of a square of area 1 a semicircle is drawn so that the side of the square is the diameter of the semicircle. Inside the square but outside the semicircle, a whole circle is inscribed so that it is tangent to two sides of the square and tangent to the semicircle. What is the radius of the circle?

A. 1/8 B. 2/9 C. $\sqrt{5}/8$
D. $2-\sqrt{3}$ E. $\sqrt{5}-2$

23. Simplify $\frac{2}{\log_4 2000}+\frac{3}{\log_5 2000}$

A. 1/6 B. 1/4 C. 1
D. 23/6 E. $\log_{20} 6$

24. Suppose $a^{\log_3 7}=27$. Then, what is $a^{(\log_3 7)^2}$

A. 54 B. 243 C. 343
D. 729 E. 2401

25. What is the next number in this sequence: 6, 42, 7, 12, 48, 16, 18, __ (Note – this problem comes from a 1960's Florida College Prep Exam)?

A. 54 B. 36 C. 24
D. 18 E. 9