CALIFORNIA STATE UNIVERSITY, BAKERSFIELD

Lee Webb Math Field Day 2018

Team Medley, Junior-Senior Level

Each correct answer is worth ten points. Answers require justification. Partial credit may be given. Unanswered questions are given zero points.

You have 50 minutes to complete the Exam. When the exam is over, give only one set of answers per team to the proctor. Multiple solutions to the same problem will invalidate each other.

Elegance of solutions may affect score and may be used to break ties.

All calculators, cell phones, music players, and other electronic devices should be put away in backpacks, purses, pockets, etc. Leaving early or otherwise disrupting other contestants may be cause for disqualification.

1. Show that there is no pair of perfect squares that differ by 2018.

2. Let S_0 be the set $\{0,1,2,3,...99\}$, S_1 be the set with the next 100 natural numbers, etc. How many of the sets $S_{0,}S_1,S_2,...,S_{999}$ do not contain perfect squares?

3. Suppose f is a function such that $f(3)=2018$ and $f(710)=11$. Prove that f can not be a polynomial with integer coefficients, of any degree.

4. In triangle ABC, D is on BC and E is on AC. AD and BE intersect at F. Triangles AEF, BDF, and ABF have areas 3, 7, and 7 respectively. What is the area of the triangle ABC?

5. For how many ordered pairs of relatively prime integers (a,b) is $\frac{5a}{b}+\frac{7b}{9a}$ an integer?

6. The shortest side of a triangle with angle measurements 36, 72, 72 degrees has length 1. What is the length of one of the other sides?