CALIFORNIA STATE UNIVERSITY, BAKERSFIELD
Lee Webb Math Field Day 2018
Individual Medley, Junior-Senior Level

Your answers to these questions should be on the side of the answer sheet that has answer spaces A, B, C, D, E (NOT 1, 2, 3, 4, 5). On the answer sheet you should write your name, school name, level (Junior-Senior), and Division (your proctor should have a list of which schools are in which divisions).

For each of the following questions, blacken the appropriate circle on the answer sheet. Each correct answer is worth four points. **One point is deducted for each incorrect answer.** An unanswered question is given zero points. Note that random guessing may adversely affect your score.

You have 50 minutes to complete the examination. If you finish early, review your answers. When the exam is over, give your answer sheet to the proctor.

All calculators, cell phones, music players, and other electronic devices should be put away in backpacks, purses, pockets, etc. Leaving early or otherwise disrupting other contestants may be cause for disqualification.

1. Which of the following functions is odd?

   A. sine B. cosine C. logarithm
   D. exp E. abs. value

2. What is the maximum number of 4” x 6” tiles that can be placed on a 4' x 6' floor without overlapping?

   A. 46 B. 64 C. 100
   D. 144 E. 156

3. Evaluate $\sum_{k=10}^{50} (k-7)$.

   A. 920 B. 943 C. 946
   D. 1037 E. 1253

4. Jose, Fatima, and Xavi are making zongzi to get ready for Tibaddi. To make 1 zongzi, Jose needs 2 hours, Fatima needs 3 hours, and Xavi needs 4 hours. If they are all working, how many hours will it take them to make 130 zongzi?

   A. 150 B. 144 C. 130
   D. 120 E. 90

5. Solve $e^x + e^{2x} = 6$

A. $\log 3 - \ln 2$ B. $\frac{\ln 4}{\ln 2}$ C. $\frac{\ln 2}{\ln 3}$

D. $\ln 2$ E. $\ln 3$

6. How many points of intersection are there on the graphs of $y = x^2 - 6x$ and $y = \frac{6}{x} - 11$?

A. 0 B. 1 C. 2

D. 3 E. 4

7. Triangle ABC is isosceles with AB=BC and angle B has measure 30 degrees. D is a point on BC, between B and C, such that triangle ADC is also isosceles. What is the sum of all the possible degree measures of angle BAD?

A. 45 B. 57.5 C. 67.5

D. 142.5 E. ∞

8. There are three non-overlapping squares, A, B, and C. A and B each have one side on the x-axis; C has only one corner on the x-axis. A and B each share one corner with C. A has a side length of 3; B has a side length of 4. What is the sum of the areas of A, B, and C?

A. 50 B. 64 C. 72

D. 84 E. $36\sqrt{5}$

9. The points (5,-4) and (-7,8) are symmetric with respect to a line L. What is the x-intercept of L?

A. -5 B. -3 C. 1

D. 4/9 E. 3

10. To the nearest square inch, what is the area of a 50-inch TV? (diagonal= 50, aspect ratio 16:9)

A. 1444 B. 1000 C. 1068

D. 1080 E. 1480

11. Circle O is centered at the origin and passes through point R at (1,0). Point P is on the circle such that angle POR is 60 degrees. A line tangent to the circle at P passes throught the x-axis at piont T. What is the measure of angle PTO, in degrees?

A. 15 B. 30 C. 45

D. 60 E. Not enough information given

12. A cubic room has side length 3. An ant one unit above a floor corner would like to travel to the opposite floor corner. What is the length of the shortest path it can take if it must always be on a surface (wall, floor, or ceiling)?

A. 5 B. 6 C. $4\sqrt{2}$

D. $4\sqrt{3}$ E. $5\sqrt{2}/3$

13. What is the cosine of the acute angle formed when a line with slope 2 intersects a line with slope -1?

A. $\frac{\sqrt{5}}{5}$ B. $\frac{\sqrt{10}}{10}$ C. $\frac{\sqrt{5}}{10}$

D. $\frac{\sqrt{10}}{5}$ E. $\frac{3\sqrt{10}}{10}$

14. In these four equations,

$$3a+4b=2c+3$$
$$3b+4c=2d+4$$
$$3c+4d=2a+1$$
$$3d+4a=2b+2$$

what is the sum of the solutions for *a*, *b*, *c*, and *d?*

A. 0 B. 2 C. 10/7

D. 10 E. 23

15. Let *r* be the proportion of red balls in a ball pit, and *b* be the proportion of blue balls. Also suppose $9r^2b=1-r$. If there are balls of both colors and no other colors, then what is the probability that a randomly chosen ball is red?

A. 1/10 B. 1/4 C. 1/3

D. $\frac{\sqrt{2}}{5}$ *E.* $\frac{2}{3}$

16. Rectangle ABCD is inscribed in a circle with radius 10. Angle CAB is 15 degrees. What is the area of the rectangle?

A. 100 B. $200\tan 15^o$ C. $200\sin 15^o$

D. $100\cos 15^o$ E. $100(\sqrt{6}-\sqrt{2})$

17. Let $\theta = 2\pi/5$, $A = \tan\theta$, $B = sec\,\theta$, $C = A + B$, $D = A - B$, and $E = CD$. Evaluate E ?

A. -4 B. -1 C. 1/4

D. 120/17 E. $2\sqrt{5}$

18. Two intrepid students each dig a hole in the Sahara Desert, 20 feet deep and 100 miles apart, on the equator. From the bottom of the hole, each student records when the sun is directly overhead. Later, they determine that there was a six-minute difference between the two times. With this information, (and assuming the earth is spherical), what should they conclude the circumference of the earth to be (in miles)?

A. 8000π B. 9000π C. 23890

D. 24000 E. 25000

19. Three boys and three girls are paired up randomly to do science projects. Which of the following is closest to the probability that each pair will be one boy and girl?

A. 20% B. 25% C. 33%

D. 40% E. 17%

20. Which of the following is closest to the number of decimal digits in 2^{100} ?

A. 15 B. 20 C. 25
D. 30 E. 35

21. A parallelogram has diagonals with lengths of 12 and 18. The diagonals meet at a 30 degree angle. What is the area of the parallelogram?

A. 48 B. 72 C. 216
D. 54 E. 108

22. Given the system of equations:

$$x^2+y+z^3=20$$
$$-x^2+y+z^3=12$$
$$-x+y^2-2z^3=18$$
$$x^2-y+z^3=8,$$

find the value of $x+y$.

A. 4 B. $12\sqrt{3}$ C. 13
D. $\dfrac{\sqrt{3}}{(5-\sqrt{3})^2}$ E. 8

23. Which of the following equals
$2(\log_2(5)\log_7(3)\log_9(16)\log_5(7))-(\log_9(7)\log_{16}(9)\log_5(2)\log_7(5))$

A. ln(15)/ln(7) B. 15/4 C. ln(63)/ln(10)
D. 23/6 E. 10

24. A man is crossing a railroad bridge and is 4/7 of the way across when he sees a train is coming. Whether he runs towards or away from the train, he has just enough time to get to the end of the bridge at the same time as the train. He can run 17 kilometers per hour. How fast, rounded off to the nearest kilometers per hour, is the train going?

A. 34 B. 66 C. 89

D. 107 E. 119

25. Which of the following polar equations represents a hyperbola?

A. $r = 3\sin\theta$ B. $r = \frac{6}{\sin\theta - 5\cos\theta}$ C. $r^2 = \frac{3}{4\sin\theta + 5\cos\theta}$

D. $r = sec\,\theta\tan\theta$ E. $r^2 = 7\,sec\,\theta\,csc\,\theta$