

CALIFORNIA STATE UNIVERSITY, BAKERSFIELD
Lee Webb Math Field Day 2018
Individual Medley, Freshman- Sophomore Level

Your answers to these questions should be on the side of the answer sheet that has answer spaces 1, 2, 3, 4, 5 (NOT A, B, C, D, E). On the answer sheet you should write your name, school name, level (Freshman-Sophomore), and Division (your proctor should have a list of which schools are in which divisions).

For each of the following questions, blacken the appropriate circle on the answer sheet. Each correct answer is worth four points. **One point is deducted for each incorrect answer.** An unanswered question is given zero points. Note that random guessing may adversely affect your score.

You have 50 minutes to complete the examination. If you finish early, review your answers. When the exam is over, give your answer sheet to the proctor.

All calculators, cell phones, music players, and other electronic devices should be put away in backpacks, purses, pockets, etc. Leaving early or otherwise disrupting other contestants may be cause for disqualification.

1. Two parabolas have distinct but parallel axes of symmetry. Which of the following statements could be true:
- I. The parabolas do not intersect.
 - II. The parabolas intersect in exactly one point.
 - III. The parabolas intersect in exactly two points.
 - IV. The parabolas intersect in more than two points.

1. I & II 2. II & III 3. III & IV
4. I, II, & III 5. All of them

2. A line is given by the equation $\frac{x}{6} + \frac{y}{10} = 3$. What is the sum of the x and y intercepts of the line?

1. 10 2. 16 3. 26
4. 28 5. 48

3. Alex's team won 12% of its games. The team played more than 50 games, but less than 100. How many games did the team play?

1. 60 2. 65 3. 70
4. 75 5. 77

4. Two lines both have y-intercept 10. The two x-intercepts are 20 and 24. What is the area of the triangle formed by these three points?

1. 20 2. 25 3. 30
4. 40 5. 48

5. Triangle ABC is isosceles with $AB=BC$ and angle ABC has measure 30 degrees. Point D is inside the triangle such that triangle ADC is also isosceles, with $AD = DC$ and angle ADC has measure 110 degrees. What is the degree measure of angle BAD

1. 30 2. 42 3. 55
4. 45 5. 40

6. Because of traffic, Joe was only able to drive an average of 42 miles per hour on the trip to a conference. He was able to average 63 miles per hour on the return trip and it took 1 hour less time. How many hours did it take Joe to drive home from the conference?

1. 1 2. 2 3. 3
4. 4 5. 5

7. What is the minimum number of diagonals of a (not necessarily convex) hexagon that are entirely within the interior of the hexagon.

1. 1 2. 2 3. 3
4. 4 5. 9

8. Circle O has its center at $(0, 0)$ and passes through point R at $(1, 0)$. Point P is on the circle such that angle POR is 60 degrees. A line tangent to the circle at P passes through the x-axis at point T. What is the measure of angle PTO, in degrees?

1. 15 2. 30 3. 45
4. 60 5. Cannot be
determined from the
given information

9. In triangle ABC, angles A and B measure 50 and 70 degrees, respectively. I is the incenter of the triangle. What is the sum of the measures of angles ABI, BCI, and CAI?

1. 50 2. 60 3. 70
4. 80 5. 90

10. A sequence is defined recursively by $x_1=7$ and $x_{n+1}=-2x_n+21$ for all n greater than 1. What is the value of x_{15} ?

1. -17 2. 0 3. 23
4. 7 5. 14

11. To the nearest square inch, what is the area of a 50-inch TV? (diagonal= 50, aspect ratio 16:9)

1. 144 2. 1000 3. 1068
4. 1080 5. 1480

12. The students in Ella's class are split into 10 groups of 3. But no matter how the teacher makes the groups, on days when the orchestra students leave, there is at least one group completely missing. What is the least number of students that could be in the orchestra?

1. 15 2. 17 3. 20
4. 21 5. 22

13. Inside of square ABCD is another square EFGH. Rays EF, FG, GH, HE go through points C, D, A, B, respectively. Furthermore, the area of ABCD is 40. The lengths of FC, GD, HA, and EB are all equal to 2. What is the area of EFGH?
1. 4 2. 6 3. 12
4. 14 5. 16
14. What is the area of a triangle with vertices at (1,4), (4,9), and (9, 16).
1. 2 2. 6 3. $8\sqrt{5}$
4. 7.5 5.
15. ABCDEFGH is a regular octagon with side length 6. What is the area inside both quadrilaterals ABEF and CDGH?
1. $18\sqrt{2}$ 2. $24\sqrt{2}$ 3. $24\sqrt{6+4\sqrt{2}}$
4. $48-8\sqrt{2}$ 5. 36
16. What is the sum of the even squares that are less than 101, minus the sum of the odd squares that are less than 101?
1. 9 2. 25 3. 55
4. 66 5. 69
17. There are three non-overlapping squares, A, B, and C. A and B each have one side on the x-axis; C has only one corner on the x-axis. A and B each share one corner with C. A has a side length of 3; B has a side length of 4. What is the sum of the areas of A, B, and C?
1. 50 2. 64 3. 72
4. 80 5. $36\sqrt{5}$

18. A cubical room has side length 3. An ant one unit above a floor corner would like to travel to the opposite floor corner. What is the length of the shortest path it can take if it must always be on a surface (wall, floor, or ceiling)?

1. 5 2. 6 3. $3\sqrt{3}$
4. $6\sqrt{3}$ 5. $5\sqrt{2}/3$

19. Three boys and three girls are paired up randomly to do science projects. Which of the following is closest to the probability that each pair will be one boy and girl?

1. 20% 2. 25% 3. 33%
4. 40% 5. 17%

20. What is the area of the figure described by the graph of the equation $|x|+|y|<10$?

1. 100 2. 200 3. $100\sqrt{2}$
4. 400 5. $50\sqrt{10}$

21. In these four equations,

$$3a+4b=2c+3$$

$$3b+4c=2d+4$$

$$3c+4d=2a+1$$

$$3d+4a=2b+2$$

what is the sum of the solutions for a , b , c , and d ?

1. 0 2. 2 3. $7/10$
4. 10 5. $120/17$

22. A man is crossing a railroad bridge and is $2/3$ of the way across when he sees a train is coming. Whether he runs towards or away from the train, he has just enough time to get to the end of the bridge at the same time as the train. He can run 20 kilometers per hour. How fast, rounded off to the nearest kilometers per hour, is the train going?

1. 30 2. 40 3. 60
4. 68 5. 73

23. A fair six-sided die has 2 red sides, 2 blue sides, and 2 yellow sides. If the die is rolled three times, what is the probability that each color is rolled once?

1. $1/6$ 2. $2/9$ 3. $1/3$
4. $4/9$ 5. $5/12$

24. Let S be the set of the first 2018 multiples of 20 and T be the set of the first 2018 multiples of 18. How many numbers are in both sets?

1. 201 2. 210 3. 218
4. 224 5. 264

25. Given the system of equations:

$$\begin{aligned}x^2 + y + z^3 &= 20 \\ -x^2 + y + z^3 &= 12 \\ -x + y^2 - 2z^3 &= 18 \\ x^2 - y + z^3 &= 8,\end{aligned}$$

find the value of $x + y$.

1. 4

2. $\frac{\sqrt{2}}{(1+\sqrt{2})^2}$

3. $3 - 2\sqrt{2}$

4. -6

5. 8