CALIFORNIA STATE UNIVERSITY, BAKERSFIELD

Lee Webb Math Field Day 2017

Team Medley, Junior-Senior Level

Each correct answer is worth ten points. Answers require justification. Partial credit may be given. Unanswered questions are given zero points.

You have 50 minutes to complete the Exam. When the exam is over, give only one set of answers per team to the proctor. Multiple solutions to the same problem will invalidate each other.

Elegance of solutions may affect score and may be used to break ties.

All calculators, cell phones, music players, and other electronic devices should be put away in backpacks, purses, pockets, etc. Leaving early or otherwise disrupting other contestants may be cause for disqualification.

1. Let ABC be a triangle and D be on side BC. Further, let segments AB, AC, AD, BD, and CD have lengths *c*, *b*, *d*, *m*, and *n*. Prove that these lengths satisfy the following formula:

$$(m+n)(d^2+mn)=(b^2m+c^2n)$$

2. One morning, Ella prepared 21 snowballs to throw at a tree for target practice. Daniel came out and said "I could hit that tree with all the snowballs even if I threw more than one at a time." "Well let's see," said Ella. She scooped up a random number of balls (between 1 and 20 of course). Daniel threw them and hit the tree, and Ella scooped up a random number of the remaining snow balls, and this process was repeated until all the snowballs were gone. What is the probability that on different throws Daniel threw 5, 5, 5, 3, and 3 snowballs, in any order?

3. In triangle ABC, we have AB=1, BC=2. Also, point D is on side CA such that BD =1 and ABD is a right angle. What is the area of triangle BCD?

4. If the numbers 1, 2, 3, …, 1024 are written out in binary notation, with no leading zeros, how many zeros are used?

5. The numbers 1, 2, 3 can be partitioned into non-empty subsets in 5 ways. Specifically, the partitions are (1)(2)(3), (12)(3), (13)(2), (23)(1), (123). How many ways can the numbers 1, 2, 3, 4, 5, 6 be partitioned?

6. The number of ways of writing 8 as a sum of at most 4 positive integers is equal to the number of ways of writing 8 as a positive integers that are all less than or equal to 4. Show that this is also true of the number 64.