CALIFORNIA STATE UNIVERSITY, BAKERSFIELD

Lee Webb Math Field Day 2017

Team Medley, Freshman-Sophomore Level

Each correct answer is worth ten points. Answers require justification. Partial credit may be given. Unanswered questions are given zero points.

You have 50 minutes to complete the Exam. When the exam is over, give only one set of answers per team to the proctor. Multiple solutions to the same problem will invalidate each other.

Elegance of solutions may affect score and may be used to break ties.

All calculators, cell phones, music players, and other electronic devices should be put away in backpacks, purses, pockets, etc. Leaving early or otherwise disrupting other contestants may be cause for disqualification.

1. Suppose a quadrilateral is given that has side lengths *a, b, c, d* (in order) and that this quadrilateral has an inscribed circle. Prove that $a+c = b+d$.

2. Ella has chosen five distinct numbers from among 1, 2, 3, 4, 5, 6, 7. She says, “Daniel – if I told you the product of my numbers, you still could not figure out whether the sum of my numbers is even or odd.” Daniel replied “In that case, I already know the product of your numbers.” Explain how Daniel could already know this and what the product of Ella's numbers is.

3. One morning, Ella prepared 10 snowballs to throw at a tree for target practice. Daniel came out and said “I could hit that tree with all the snowballs even if I threw more than one at a time.” “Well let's see,” said Ella. She scooped up a random number of balls (between 1 and 10 of course). Daniel threw them and hit the tree, and Ella scooped up a random number of the remaining snow balls, and this process was repeated until all the snowballs were gone. What is the probability that on different throws Daniel threw 3, 3, 2, and 2 snowballs, in any order?

4. Let ABC be a triangle and D be the midpoint of side BC. Further, let segments AB, AC, AD, and BD have lengths *m, n, l, a*. Determine a formula for *l* in terms of *m, n,* and *a*.

5. The numbers 1, 2, 3 can be partitioned into non-empty subsets in 5 ways. Specifically, the partitions are (1)(2)(3), (12)(3), (13)(2), (23)(1), (123). How many ways can the numbers 1, 2, 3, 4, 5 be partitioned?

6. The number of ways of writing 8 as a sum of at most 3 positive integers is equal to the number of ways of writing 8 as a positive integers that are all less than or equal to 3. Show that this is also true of the number 24.