CALIFORNIA STATE UNIVERSITY, BAKERSFIELD
Lee Webb Math Field Day 2017
Individual Medley, Junior-Senior Level

Your answers to these questions should be on the side of the answer sheet that has answer spaces A, B, C, D, E (NOT 1, 2, 3, 4, 5). On the answer sheet you should write your name, school name, level (Junior-Senior), and Division (your proctor should have a list of which schools are in which divisions).

For each of the following questions, blacken the appropriate circle on the answer sheet. Each correct answer is worth four points. **One point is deducted for each incorrect answer.** An unanswered question is given zero points. Note that random guessing may adversely affect your score.

You have 50 minutes to complete the examination. If you finish early, review your answers. When the exam is over, give your answer sheet to the proctor.

All calculators, cell phones, music players, and other electronic devices should be put away in backpacks, purses, pockets, etc. Leaving early or otherwise disrupting other contestants may be cause for disqualification.

1. What is the minimum value of the function $f(x)=2(x-4)(x+6)+25$

A. -25 B. -5 C. 0
D. 1 E. 2

2. Using the digits 2, 0, 1, 7, each once, how many four-digit number can be made – leading 0 not allowed?

A. 10 B. 12 C. 18
D. 17 E. 15

3. Four boys and four girls are going to play a game that needs two teams of 4 each. If the teams are chosen randomly, what is the probability that the game will end up being boys against girls?

A. 1/2 B. 1/8 C. 1/12
D. 1/24 E. 1/35

4. A triangle has perimeter 20 and each side's length is a natural number. What is the largest possible value of any of its side lengths.

A. 12 B. 11 C. 10
D. 9 E. 8

5. At Pete's Wonderbank, which compounds continuously, account balances triple every 8 years. How many years will it for an account to merely double?

A. 5 B. $8\frac{\ln 3}{\ln 2}$ C. $8\frac{\ln 2}{\ln 3}$

D. $8e^{3/2}$ E. $8\ln\frac{3}{2}$

6. What is the equation of the parabola with directrix $x=-4$ and focus at $(2,0)$?

A. $12y^2=x+1$ B. $x=3y^2-1$ C. $x=y^2/12-1$

D. $y^2=12x+1$ E. $12y^2+1=x$

7. A square has area 100. One vertex of an equilateral triangle is at a corner of the square and the other two vertices of the triangle are on the sides of the square. What is the perimeter of the triangle.

A. 24 B. $60-30\sqrt{3}$ C. $30(\sqrt{6}-\sqrt{2})$

D. $12\sqrt{2}+6\sqrt{3}$ E. $\frac{24}{\sqrt{6}+4\sqrt{3}}$

8. A pizza is cut into 6 equal pieces. Then the curved edge of each piece is cut off to make each piece a triangle. To the nearest whole number, what percentage of the pizza is removed?

A. 23 B. 14 C. 20

D. 17 E. 5

9. The points (5,-2) and (-7,8) are symmetric with respect to a line L. What is the y-intercept of L?

A. 5/18 B. 21/5 C. 21/4
D. 4/9 E. 12

10. How many of the factors of 10800 are squares?

A. 4 B. 6 C. 8
D. 10 E. 12

11. How many 4 digit numbers are there that contain at least one of the digits 2, 0, 1, 7 – no leading zeros allowed?

A. 6000 B. 8445 C. 8103
D. 7704 E. 7548

12. What is the largest factor of 10! that is a square?

A. 3600 B. 14400 C. 25600
D. 129600 E. 518400

13. No positive integer has the property that when multiplied by 17, the result is 50 less than the number squared. Which positive integer comes closest to having this property?

A. 17 B. 18 C. 19
D. 20 E. 22

14. Assuming $i^2=-1$, simplify $\frac{29+31i}{7+2i}$.

A. $5+3i$ B. $7-3i$ C. $4-7i$

D. $4+5i$ E. $3+5i$

15. Point T is on a circle and point P is such that line PT is tangent to the circle and segment PT has length 12. Points A and B are on the circle so that chord AB has length 9 and so that A, B, P are collinear. What is the shortest possible value of segment AP?

A. 8 B. 9 C. 10

D. $\frac{\sqrt{657}-9}{2}$ *E.* $\frac{\sqrt{4213}+16}{144}$

16. In rectangle ABCD, AB has length 10, BC has length 6. Side AB is also the diameter of a semicircle that is inside the rectangle. A circle is inscribed so that it is tangent to the semicircle and to sides BC and CD. What is the radius of the circle?

A. 1 B. 2 C. $16-2\sqrt{55}$

D. $6-3\sqrt{2}$ E. $\frac{15-\sqrt{10}}{2}$

17. A right triangle has perimeter 40 and the shortest side has length 8. What is the altitude from the right angle to the hypotenuse?

A. 6 B. 7 C. $2\sqrt{30}$

D. 120/17 E. $4\sqrt{2}$

18. Given that $x^2+y^2=14x+16y-113$, what is the value of $x\cdot y$?

A. -24 B. 24 C. 56

D. -36 E. 96

19. Rip Winkle has 17 sons and no daughters. Some of his sons have 17 sons and no daughters, and the rest have no children. When they all got together for a family portrait, there were 188 people in the picture. How many of these people do not have children?

A. 127 B. 147 C. 177

D. 211 E. 217

20. A square is inscribed in a circle. On each side of the square, two smaller, congruent, squares are inscribed so that one side of each is part of the original square and so that they share one side with each other and each of these smaller squares has one vertex on the circle. What is the ratio of the area of one of the smaller squares to the area of the original square?

A. 1/12 B. 1/8 C. $\frac{2-\sqrt{3}}{8}$

D. $\frac{9-3\sqrt{2}}{16}$ E. $\frac{12+\sqrt{2}}{64}$

21. A parallelogram has diagonals with lengths of 12 and 18. The diagonals meet at a 30 degree angle. What is the area of the parallelogram?

A. 48 B. 72 C. 216

D. 54 E. 108

22. For how many values of x in the range $0 \le x \le \pi$ does $\sin 5x = \sin 12x$?

A. 5 B. 12 C. 13

D. 17 E. 24

23. A pyramid has a base that is a regular hexagon with side length 1. The apex is 1 unit directly above the center of the base. What is the surface area of the pyramid, not counting the area of the base?

A. $3\sqrt{7}/2$ B. 6 C. 12

D. 10 E. $6+3\sqrt{3}$

24. Consider the following system of equations:

$$a+b+c-d=10$$
$$a+b-c+d=11$$
$$a-b+c+d=12$$
$$-a+b+c+d=13$$

What is the value of $a+b+c$?

A. 18 B. 23 C. 33/2

D. 61/3 E. 54/5

25. Define $n!!$ to be equal to $n(n-2)(n-4)\cdots a$, where $a = 1$ or 2 depending on whether n is odd or even. Also define $\left(\!\binom{n}{k}\!\right) = \frac{n!!}{k!! \cdot (n-k)!!}$. Evaluate $\left(\!\binom{12}{6}\!\right) - \left(\!\binom{9}{5}\!\right)$.

A. 1/10 B. 97/8 C. 137/24

D. 23 E. 108/5