45th Lee Webb Math Field Day

California State University, Bakersfield
Department of Mathematics

March 12, 2016

Varsity Math Bowl

Varsity Math Bowl Round 1 Sample Question

Today is March 12. How many positive divisors does 12 have?

Varsity Math Bowl Round 1 Question 1

What is the least common multiple of 12, 15, 18?

Varsity Math Bowl Round 1 Question 2

What is the degree of the polynomial

$$(x-1)(x^2-2)(x^3-3)(x^4-4)?$$

Varsity Math Bowl Round 1 Question 3

What is x-coordinate of the point of intersection of the lines

$$3x + 10y = 24$$

$$1x + 5y = 9$$

Varsity Math Bowl Round 1 Question 4

If three numbers are chosen randomly from the set

$$\{1, 3, 5, 7, 9, 11, 13\},$$

what is the probability that their sum will be less than 10? (Answer needs to be a fraction in reduced form.)

Varsity Math Bowl Round 1 Question 5

Suppose $\frac{1}{x} + \frac{1}{y} = 50$ and $\frac{1}{xy} = 20$, what is $x + y$?

Varsity Math Bowl Round 1 Question 6

How many two digit primes contain the digit 2?

Varsity Math Bowl Round 1 Question 7

The first term of a geometric sequence is 4. The fourth term is 108. What is the third term?

Varsity Math Bowl Round 1 Question 8

A geologist has four rocks and a balance scale. Rock 1 is heavier than Rock 2. Together, Rock 1 and Rock 2 exactly balance Rock 3 and Rock 4. Together, Rock 1 and Rock 3 are lighter than Rock 2 and Rock 4. From lightest to heaviest, list the order of the rocks. (e.g. you might enter 1234).

Varsity Math Bowl Round 1 Question 9

How many three-digit numbers are palindromes (read the same forward and backward)?

Varsity Math Bowl Round 1 Question 10

The average of 20 numbers is 16. The average of 16 other numbers is 20. What is the average of all the numbers?

(answer needs to be a reduced fraction.)

Round 2

Varsity Math Bowl Round 2 Sample Question

What is the cube root of the smallest positive perfect cube that ends in 3?

Varsity Math Bowl Round 2 Question 1

What is the y-coordinate of the focus of the parabola

$$y = \frac{1}{20}x^2$$

Varsity Math Bowl Round 2 Question 2

Solve for x:

$$x^3 - 5x^2 + 3x - 15 = 0$$

Varsity Math Bowl Round 2 Question 3

Let
$f(x) = (x-1)(x-2)(x-3)(x-4)(x-5)$.
Evaluate $f(8)$

Varsity Math Bowl Round 2 Question 4

Two sides of a triangle are both length 10 and the angle between them measures 30 degrees. What is the area of the triangle?

Varsity Math Bowl Round 2 Question 5

Reduce the fraction to simplest form:

$$\frac{9 \cdot 4! \cdot 6! \cdot 8! \cdot 5}{5! \cdot 7! \cdot 9!}$$

Varsity Math Bowl Round 2 Question 6

This year, 2016, is divisible by 7. Altogether, how many years this century are divisible by 7?

Varsity Math Bowl Round 2 Question 7

$AB = 7$, $BC = 24$, and $CA = 25$. In triangle ABC, what is the length of the altitude from angle B?

Varsity Math Bowl Round 2 Question 8

A line goes through the origin and makes an acute angle θ with the y-axis. If $\tan\theta = 3/7$, what is the slope of the line?

Varsity Math Bowl Round 2 Question 9

Consider the sequence of numbers

$$1, 2, 2, 3, 3, 3, 3, 4, 4, 4$$

$$4, 4, 4, 4, 4, 5, 5..., 23, 24.$$

What is the median of the numbers in this sequence?

Varsity Math Bowl Round 2 Question 10

Simplify:

$$\frac{18+15-12-9+6-3}{24+20-16-12+8-4}$$

Round 3

Varsity Math Bowl Round 3 Sample Question

2016 can be written as the sum of three consecutive numbers. What is the smallest of these numbers?

Varsity Math Bowl Round 3 Question 1

What is the largest prime divisor of 2016?

Varsity Math Bowl Round 3 Question 2

How many pairs of real numbers (a,b) satisfy the equation

$$a^2b^2 - 3b^2 - 4a^2 + 12 = 0$$

Varsity Math Bowl Round 3 Question 3

A square ABCD has side length 17. Inside it, another square, PQRS, touches each side of ABCD, so that $AP = BQ = CR = DS = 5$. A circle inside PQRS is tangent to all sides of PQRS. The circumference of this circle is what multiple of π?

Varsity Math Bowl Round 3 Question 4

Evaluate:

$$\lim_{x \to 2} \frac{x^2 - 4}{x^3 - 8}$$

Varsity Math Bowl Round 3 Question 5

What is the sum of all the positive solutions to the equation:

$$\sqrt{x}^3 = \sqrt[3]{x}^2$$

Varsity Math Bowl Round 3 Question 6

Simplify:

$$\cos(\frac{17\pi}{87})\cos(\frac{4\pi}{29}) - \sin(\frac{17\pi}{87})\sin(\frac{4\pi}{29})$$

Varsity Math Bowl Round 3 Question 7

What is the 4-digit number with digits 3,4,5, and 6, that is divisible by all odd primes less than 13 ?

Varsity Math Bowl Round 3 Question 8

Alex lives 60 miles west of Jo. If they leave their houses at the same time and both drive east, Alex catches up to Jo in 6 hours. If they leave at the same time and go north for 8 hours, then how far apart will they be?

Varsity Math Bowl Round 3 Question 9

Solve:

$$\log_4 x = \log_8 512$$

Varsity Math Bowl Round 3 Question 10

What is the absolute value of the largest negative solution of

$$\cos(\pi x/8) = 1/2$$

Round 4

Varsity Math Bowl Round 4 Sample Question

After shuffling a standard deck of cards (no jokers), what is the probability that the top two cards are both aces?

Varsity Math Bowl Round 4 Question 1

Write

$$\frac{1}{4+\frac{1}{5+\frac{1}{6}}}$$

as a fraction a/b in lowest terms.

Varsity Math Bowl Round 4 Question 2

A standard deck of cards (no jokers) is shuffled. What is the probability that the 7th and 11th cards are differenct suits?

Varsity Math Bowl Round 4 Question 3

What is the absolute value of the x-coordinate of the highest point of the graph of $y = 2x^3 - 3x^2 - 36x + 3$ that lies to the left of the y-axis?

Varsity Math Bowl Round 4 Question 4

How many of the first 15 primes leave a prime remainder when divided by 5?

Varsity Math Bowl Round 4 Question 5

When $x = 3$ what is the slope of the cubic curve

$$y = (x - 1)(x - 2)(x - 3)$$

Varsity Math Bowl Round 4 Question 6

Adam, Beth, Chris, Dave, and Ella want to sit together in one row at the movie. Adam won't sit next to Beth or Dave. Beth won't sit next to Ella. Chris and Ella have to sit next to each other because they share popcorn. How many sitting arrangements satisfy all these conditions?

Varsity Math Bowl Round 4 Question 7

Evaluate:

$$\lim_{x \to 0} \frac{1 - \cos(x^2)}{e^{x^4} - 1}$$

Varsity Math Bowl Round 4 Question 8

Two sides of a triangle have lengths 7 and 9. The angle θ between these sides is such that $\cos\theta = \frac{5}{21}$. What is the length of the third side of the triangle?

Varsity Math Bowl Round 4 Question 9

What is the maximum value of the function $f(x) = 8 + 4x^2 - 2x^4$?

Varsity Math Bowl Round 4 Question 10

The fifth derivative of a function is $5 \sin x + x \cos x$. What is the tenth derivative of the function, evaluated at π?

Varsity Math Bowl Round 4 Bonus Question

Evaluate:

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^2}{n^3}$$

The End

Please be patient while we calculate the scores.

Closing Ceremony to commence shortly