CALIFORNIA STATE UNIVERSITY, BAKERSFIELD
Lee Webb Math Field Day 2015
Team Medley, Junior-Senior Level

Each correct answer is worth ten points. Answers require justification. Partial credit may be given. Unanswered questions are given zero points.

You have 50 minutes to complete the Exam. When the exam is over, give only one set of answers per team to the proctor. Multiple solutions to the same problem will invalidate each other.

Elegance of solutions may affect score and may be used to break ties.

All calculators, cell phones, music players, and other electronic devices should be put away in backpacks, purses, pockets, etc. Leaving early or otherwise disrupting other contestants may be cause for disqualification.

1. Starting at the origin, Peter Rabbit needs to hop to the point at (5,5). Peter always hops either in the positive x or y direction and each hop has length 1. How many paths can Peter take – if each path must go through at least one of the following points: (2,3), (2,4), (3,3), (3,4), (4,3), (4,4).

2. Suppose ABC is a triangle with side-lengths 5, 12, and 13. Suppose A is the smallest of the angles and C is the largest. Consider the zig-zag path that starts at C and goes straight until it meets AB perpendicularly, and then goes straight back and meets AC perpendicularly, where turns and comes back to AB, and continues in this pattern, getting closer and closer to A. How long is this path before it gets to A?

3. One morning, Ella placed 4 coins on a table, all heads up. During the day, Daniel walks by the table 8 times. Each time, he randomly picks one coin and turns it over. What is the probability that at the end of the day all the coins are heads up?

4. Label the corners of an 8.5 by 11 piece of paper with the letters A, B, C, D. so that AB = CD = 11. Fold the paper so that D is on side AB and the crease goes through A. Then unfold. Fold the paper so that C is on side AB and crease goes through B. Unfold. The two creases meet at point P and continue to meet side CD at points Q and R. What is the area of triangle PQR?

5. Segment AB has length 8 and C is the midpoint. Semicircles are drawn with diameters AC and AB. A circle is tangent to both semicircles and to AB. What is the radius of this circle?

6. The numbers, 1, 2, 3, …, 2015 are written on the board. Then, 31 and 41 are erased. Then, any number that is 31 or 41 more than an erased number is also erased. This is continued until no more such numbers are left? What is the highest remaining number?