CALIFORNIA STATE UNIVERSITY, BAKERSFIELD
Lee Webb Math Field Day 2015
Individual Medley, Junior-Senior Level

For each of the following questions, blacken the appropriate circle on the answer sheet. Each correct answer is worth four points. **One point is deducted for each incorrect answer.** An unanswered question is given zero points. Note that random guessing may adversely affect your score.

You have 50 minutes to complete the examination. If you finish early, review your answers. When the exam is over, give your answer sheet to the proctor.

All calculators, cell phones, music players, and other electronic devices should be put away in backpacks, purses, pockets, etc. Leaving early or otherwise disrupting other contestants may be cause for disqualification.

1. On a standard 12-hour digital clock, what is the greatest sum of the digits possible?

A. 16 B. 17 C. 20
D. 23 E. 24

2. For a list of 8 positive integers, the mean, median, range, and mode all = 10. What is the largest possible integer in this list?

A. 15 B. 16 C. 17
D. 18 E. 19

3. One side of a triangle increases by 10%. Another side decreases by 10%. The angle between them does not change. What is the percentage change in the area of the triangle?

A. -5 B. -1 C. 0
D. 1 E. 5

4. The set of whole numbers {4,6,8,10,A,B} has range 6 and the mean and median are both 7. What is the sum of the all the possible value of of the product AB?

A. 180 B. 182 C. 200
D. 222 E. 224

5. How far is the point (10,10) from the line $3x+5y=15$?

A. 7 B. 8 C. $15/\sqrt{2}$
D. $80/\sqrt{23}$ E. $65/\sqrt{34}$

6. Suppose $\log_3\left(\frac{9x}{y}\right)=5$ and $\log_3(xy)=7$. Find $x-y$.

A. 3 B. 144 C. 216
D. 234 E. 484

7. How many of the factors of $2^5 3^6$ are squares?

A. 8 B. 10 C. 12
D. 13 E. 15

8. Suppose $700 is deposited into an account that earns 4% annual interest compounded continuously. How many years will pass before the account balance is $2100?

A. 700 B. $\frac{\ln 2100}{\ln 700}$ C. $4\ln 3$
D. $25\ln 3$ E. $120e^{-3}$

9. What are the last three digits before the long ending block of zeros of $64^{25}25^{64}$?

A. 304 B. 728 C. 625
D. 144 E. 512

10. A and B are two 5 digit numbers. Together every digit is used once. How many possible pairs (A,B) are there that maximize A+B?

A. 8 B. 10 C. 16
D. 32 E. 64

11. Suppose x is represented in base 3 by 0.12121212121212.... (repeating forever). What is x written as a fraction in base 10?

A. 12/99 B. 1/2 C. 5/9
D. 5/8 E. 2/3

12. Regular hexagon ABCDEF has side length 2. What is the area of triangle ACE?

A. $3\sqrt{3}$ B. $\frac{3\sqrt{3}}{2}$ C. $\frac{3\sqrt{6}}{4}$
D. $\frac{3\sqrt{6}}{2}$ E. 6

13. A triangle has angles that measure 45, 60, and 75 degrees. The length of the shortest side is 8 units. What is the radius of the circle that circumscribes this triangle?

A. $4\sqrt{2}$ B. $8\sqrt{2}$ C. $4\sqrt{3}$
D. $8\sqrt{3}$ E. $6\sqrt{6}$

14. Circle A has radius 1 and center at (5,0). Circle B has radius 5 and center (17,0). Line l is tangent to both circles. Assuming l does not cross the x-axis between 5 and 17, what is the x-intercept of l?

A. -2 B. -1 C. 0
D. 1 E. 2

15. What is the maximum possible value of the product xy, given that $x + 1/y = 7/2$ and $y + 1/x = 7/5$?

A. 2/5 B. 5/2 C. 5
D. 7 E. 14

16. What is the probability that a list of 5 different positive single digit numbers, whose sum is 31, will contain exactly 3 primes?

A. 1/2 B. 1/3 C. 1/4
D. 1/5 E. 31/120

17. A circle with center O has radius 10. Point P is 6 units away from O. How many chords of the circle are there that go through P and have integer length (one such chord is a diameter)?

A. 7 B. 8 C. 10
D. 12 E. 17

18. Point P is 5 units from the center of a circle with radius 7. How many chords AB does the circle have with the property that P is on AB and the lengths of PA and PB are both integers?

A. 4 B. 5 C. 6
D. 7 E. 10

19. Let $f(x)=\log_3(\log_{14}(\log_{15}(\log_9 x)))$. What is the value of c such that the largest possible domain of f is all real numbers greater than c?

A. 9^{15} B. $3\cdot 14\cdot 15\cdot 9$ C. $9^{15^{14}}$
D. 15^{14} E. 15^9

20. Square ABCD has side-length 12. Inside the square is a semicircle with diameter AB. E is on side AD such that EC is tangent to the semicircle. What is the length of CE?

A. 10 B. 15 C. $12\sqrt{3}$
D. $15\sqrt{3}$ E. 16

21. On the number line, how many numbers are five units away from their reciprocals?

A. 0 B. 1 C. 2
D. 3 E. 4

22. Two circles with radii equal to 3 and 9 are tangent to each other. What is the length of a band that is stretched around both of them?

A. $14\pi+12\sqrt{3}$ B. $15\pi+24\sqrt{3}$ C. 24π
D. $36\sqrt{3}$ E. $24\pi+6\sqrt{3}$

23. Let $i=\sqrt{-1}$ and for a complex number $\overline{a+bi}=a-bi$ is the complex conjugate of $a+bi$. Let a sequence of complex numbers $z_1, z_2, z_3, \ldots$ be defined by $z_1=1+i$ and $z_{n+1}=iz/\bar{z}$. What is the the 2015th term in the sequence?

A. 0 B. 1 C. i
D. -1 E. -i

24. Let *n* be the number of solutions of $1=csc\,2x\,sec\,2x-2\,csc\,2x+\frac{1}{2}sec\,2x$ that are in the interval $(0,2\pi)$ and let θ be the smallest solution in the interval. What is the value of $n\theta$

A. $\pi/3$ B. $2\pi/3$ C. $\pi/2$
D. π E. $5\pi/8$

25. Equilateral triangle XYZ is inscribed in equilateral triangle ABC such that X, Y, Z are on sides BC, CA, AB, respectively. Furthermore XY, YZ, ZX, are perpendicular to AC, BA, CB, respectively. What is the ratio of the area of XYZ to ABC?

A. $\frac{\sqrt{3}}{3}$ B. $\frac{1}{3}$ C. $\frac{\sqrt{3}-1}{2}$
D. $\frac{\sqrt{5}-1}{2}$ E. $\frac{1}{2}$