

# 2013 Math Bowl

## Junior Varsity Math Bowl

# 2013 Math Bowl

## Junior Varsity Math Bowl

### Round 1

# 2013 Math Bowl

Sample:

Simplify

$$\frac{3/2}{3/14}$$

# 2013 Math Bowl

## I.1 Simplify

$$\frac{\frac{1}{2} + \frac{1}{3}}{\frac{1}{4} + \frac{1}{5}}$$

# 2013 Math Bowl

1.2 What is the slope of the line given by the equation

$$x = 3y + 2$$

# 2013 Math Bowl

A hiker considers his starting point to be the origin of a Cartesian grid. He hikes NW for  $\sqrt{2}$  miles. Then turns 90 degrees to his right and hikes twice as far. Then, he turns 90 degrees to the right again and hikes three times as far as on the first portion of the hike. What is his final x coordinate (in miles)?

# 2013 Math Bowl

I.4

Simplify:

$$\frac{3}{9} \cdot \frac{4}{8} \cdot \frac{5}{7} \cdot \frac{6}{6} \cdot \frac{7}{5} \cdot \frac{8}{4}$$

# 2013 Math Bowl

I.5

Find the sum

$$163 + 167 + 171 + 175 + 179 + 183 + \dots + 203$$



# 2013 Math Bowl

I.6

What is the area of an isosceles triangle with side lengths 10, 10, 16?

# 2013 Math Bowl

I.7

Mary was 28 when she had her first child Philip. Four years later, her daughter Angela was born. How old was Mary when she was twice as old as Angela?

# 2013 Math Bowl

I.8

The midpoints of the sides of a square are connected to form another square. Likewise the midpoints of the sides of the second square are connected to form a third square. This is repeated to form a fourth square. What is the ratio of the area of the first square to the fourth square?

# 2013 Math Bowl

I.9

Solve

$$\frac{x + \frac{1}{x}}{x - \frac{1}{x}} = 2, x > 0$$

# 2013 Math Bowl

I.10

A rectangle has length 8 and width 6. Segments are drawn from each vertex to the center of the rectangle. The midpoints of these segments form another rectangle. What is the area outside the inner rectangle, but still inside the outer one?

# 2013 Math Bowl

## Junior Varsity Math Bowl

### Round 2

# 2013 Math Bowl

## Sample Question

Simplify:

$$\frac{3102 - 2013}{9}$$

# 2013 Math Bowl

II.1

Mary was 28 when she had her first child Philip. Four years later, her daughter Angela was born. How old was Mary when she was thrice as old as Angela?



# 2013 Math Bowl

II.2

The midpoints of two sides of an obtuse triangle are joined. This segment determines another triangle and a trapezoid. What is the ratio of the area of the trapezoid to the area of the original triangle?

(answer as a common fraction, in lowest terms)

# 2013 Math Bowl

II.3

Find the sum

$$1 + 3 + 5 + 7 + \cdots + 2011 + 2013$$

# 2013 Math Bowl

II.4

Ella has 10 crayons to color a map. She knows that she only needs 4 of them. How many ways can she choose the 4 colors?

# 2013 Math Bowl

II.5

Suppose

$$x+2y+3z = 10 \quad \text{and}$$

$$3x+2y+z = 30.$$

Determine  $y+2z$

# 2013 Math Bowl

II.6

The sides of a triangle have lengths 10, 10, 12. What is the length of the altitude that divides the triangle into two congruent subtriangles?

# 2013 Math Bowl

II.7

Solve

$$\frac{x + 3}{x - 3} = 3$$

# 2013 Math Bowl

II.8

The number 2013 is not prime.  
What is its largest prime factor?

# 2013 Math Bowl

II.9

In a triangle with side lengths 12, 16, 20, what is the radius of the circumscribed circle?



# 2013 Math Bowl

II. 10

Evaluate:

$$(\sqrt{2})^{2^3}$$

# 2013 Math Bowl

## Junior Varsity Math Bowl

### Round 3



# 2013 Math Bowl

III. 1

If two standard dice are rolled, what is the probability that the resulting sum is greater than 3 and less than 11?

Answer as a fraction in reduced form.

# 2013 Math Bowl

III. 2

Shelly remembers that her PIN consisted of 4 distinct odd digits. How many such PINs are there?

# 2013 Math Bowl

## III. 3

At Hal's Country Store, the bargain bin contains candles that are marked \$2.00 a piece. But there is a sign next to the bin that says “buy 3, get one free.” How much would Hal charge for 16 candles?

# 2013 Math Bowl

## III.4

A circle and three lines are drawn in the plane. What is the maximum number of points of intersection that could be determined?

# 2013 Math Bowl

## III.5

Given that

$$3x + 4y = 10$$

$$7x + 9y = 21$$

Find

$$x + y$$



# 2013 Math Bowl

III.6

Simplify

$$\frac{\frac{1}{2} + \frac{3}{4}}{\frac{5}{6} + \frac{7}{8}}$$

# 2013 Math Bowl

III.7

What is the slope of the line that goes through the two points on the parabola  $y = x^2$  that have x-coordinates 3 and 5?

# 2013 Math Bowl

III.8

A box measures 3 by 4 by 12 inches. In inches, how long is a main diagonal of the box?

# 2013 Math Bowl

III.9

A regular hexagon has perimeter 48. What is the area?

# 2013 Math Bowl

III.10

What is the tenth number in the sequence 1, 3, 6, 10, 15, ....?

# 2013 Math Bowl

## Junior Varsity Math Bowl

### Round 4

# 2013 Math Bowl

Sample Question:

Simplify

$$1111 \times 1001$$

# 2013 Math Bowl

IV.1

Let  $X$  be the sum of all the even numbers between 0 and 1000, inclusive. Let  $Y$  be the sum of all the odd numbers between 0 and 1000.

Solve for  $X-Y$ .



# 2013 Math Bowl

IV.2

How many diagonals can be drawn in a regular hexagon?

# 2013 Math Bowl

## IV.3

In a mathematics contest with ten problems, a student receives 5 points for a correct answer and is penalized 2 points for an incorrect answer. If Olivia answered every problem and her score was 29, how many correct answers did she have?

# 2013 Math Bowl

IV.4

In one week Daniel ate 91 jelly beans. Each day he ate 3 more than the previous day. How many jelly beans did he eat on the last day?

# 2013 Math Bowl

IV.5

Assume  $f(x)$  is an odd function and the point  $(-4,5)$  is on the graph of  $f$ . With the given information, there is one other point that must be on the graph. Suppose this point is  $(a,b)$ . Determine  $a-b$ .

# 2013 Math Bowl

IV.6

A set of 15 numbers is arranged in order. The difference between each pair of consecutive numbers is less than 10 and the sum of all the numbers is 3000. What is the average of the numbers?

# 2013 Math Bowl

## IV.7

A standard analog clock goes at twice the normal speed. In a 24 hour period, how many times will this clock show the correct time?

# 2013 Math Bowl

IV.8

Find the value of  $a$  that makes the following two lines perpendicular:

$$2x - 40y = 10$$

$$ax + 5y = 17$$

# 2013 Math Bowl

## IV.9

A  $4 \times 4$  magic square is filled with the numbers 1 through 16 such that each row, column, and diagonal sums to the same magic sum. What is the value of this sum?



# 2013 Math Bowl

IV.10

A line with slope 3 goes through the point  $(13, 50)$ . What is the  $y$ -intercept of this line?

# 2013 Math Bowl

## Varsity Math Bowl

# 2013 Math Bowl

## Varsity Math Bowl

### Round 1

# 2013 Math Bowl

Sample:

What is the sum of all the solutions  
to  $x/3=27/x$ ?

# 2013 Math Bowl

I.1

Simplify

$$\log_4 5 \log_5 6 \log_6 7 \log_7 8$$

# 2013 Math Bowl

I.2

Three sides of a triangle have lengths 5, 8 and  $x$ . The possible values of  $x$  have a maximum lower bound of  $a$  and a minimum upper bound of  $b$ . What is the product of  $a$  and  $b$ ?

# 2013 Math Bowl

I.3

An infinite geometric series starts with the terms: 36, -12, 4. What is the sum of the series?

# 2013 Math Bowl

I.4

How many diagonals can be drawn in a regular decagon?



# 2013 Math Bowl

I.5

Express  $.20333333\dots$  as a fraction in lowest terms.

# 2013 Math Bowl

I.6

What is the first Fibonacci number, greater than 1, that is a perfect square?

# 2013 Math Bowl

I.7

In a triangle with side lengths 12, 16, 20, what is the radius of the inscribed circle?

# 2013 Math Bowl

I.8

The eleventh digit of an eleven digit number is illegible. But it is known that the first 10 digits are 1234567890. What is the eleventh digit, if it is also known that the whole number is divisible by eleven?

# 2013 Math Bowl

I.9

For an arbitrary triangle, which of the following points is not on the Euler Line?

1) incenter

2) circumcenter

3) centroid

4) orthocenter

# 2013 Math Bowl

I.10

A cube has how many axes of symmetry?

# 2013 Math Bowl

## Varsity Math Bowl

### Round 2

# 2013 Math Bowl

Sample:

If  $3x+4y=10$

and  $4x+3y=11$

What is  $x+y$ ?



# 2013 Math Bowl

II.1

What is the number exactly half way between  $\frac{1}{4}$  and  $\frac{1}{6}$  ?

# 2013 Math Bowl

## II.2

The base six representation of a number is 235,423. Which of the following is its base 10 representation?

- 1) 20679      2) 21679      3) 20678  
4) 20779      5) 20377

(answer by entering 1, 2, 3, 4, or 5)

# 2013 Math Bowl

II.3

Mary was 28 when she had her first child Philip. Four years later, her daughter Angela was born. How old was Mary when her age was 25% more than Angela's?

# 2013 Math Bowl

II.4

Assume  $f(x)$  is an odd function with domain that includes the interval  $(-6,6)$  and that  $(-4,5)$  is on the graph of  $f(x)$ . With the given information, there are other points that must be on the graph. What is the product of the coordinates of the points that must be on the graph?

# 2013 Math Bowl

II.5

Simplify

$$\left| \overline{(1 + 3i) + (-1 - 3i)} \right|$$

# 2013 Math Bowl

II.6

Simplify

$$e^{5\ln 4}$$

# 2013 Math Bowl

Let  $U = \{1, 2, 3, 4, 5, 6, 7, 8, \dots, 100\}$  and  $A_i$  be the

multiples of  $i$  that are in  $U$ . How many elements are in the complement of the set  $A_5 \cap A_7$

# 2013 Math Bowl

II.8

A clock goes at twice the normal speed. In a 24 hour period, how many times will the minute hand of this clock point in the correct direction?



# 2013 Math Bowl

II.9

Happy makes a list of all the possible rearrangements of the digits 2013. What is the sum of all the digits on Happy's list?

# 2013 Math Bowl

II.10

The graph of a third degree polynomial goes through the points  $(0,0)$ ,  $(1,0)$ ,  $(2,0)$ , and  $(3,-6)$ . What is the  $y$ -coordinate, when  $x=-1$ ?

# 2013 Math Bowl

## Varsity Math Bowl

### Round 3

# 2013 Math Bowl

Sample:

The legs of a right triangle have lengths of 2 and 3. What is the length of the hypotenuse?

# 2013 Math Bowl

## III.1

A car is traveling at 50 mph. Its tires are 26 inches in diameter. What is the speed of a point at the top of a tire (in mph)?

# 2013 Math Bowl

In a plane, two circles each have radius equal to  $\sqrt{3}$  and the distance between the two centers is 3. What is the area of the quadrilateral determined by the two centers and the two points where the circles intersect?

# 2013 Math Bowl

III.3

If three standard dice are rolled, what is the probability that the resulting sum is less than 6?

# 2013 Math Bowl

## III.4

A perfect number is one that is the sum of its proper divisors. The lowest such number is 6. What is the next lowest?



# 2013 Math Bowl

## III.5

For each value of  $k$ , the equation

$$(x - 5 - k)^2 + y^2 = k^2$$

represents a circle. All these circles have one point in common. What is the  $x$ -coordinate of this point?

# 2013 Math Bowl

## III.6

Simplify the following, expression –  
answering as a fraction in lowest  
terms:

$$\log_{49}343 + \log_8 4 - \log_{10}0.01$$

# 2013 Math Bowl

A peach tree grows in the corner where four families' yards come together. They pick the harvest together and divide it evenly. There is one left over, which they leave for the birds. Each family has five members; they divide the fruit evenly and each family has one left over, which is put out for the birds. Each person in each family decides to share their fruit with evenly with 2 friends. Each of these divisions leaves one left over. What is the smallest number over 200 that could equal the number of peaches in the harvest?

# 2013 Math Bowl

III.8

Three of the faces of a rectangular prism have surface areas of 12, 15, and 20 sq. units. What is the volume of the prism?

# 2013 Math Bowl

III.9

What is the constant term of the polynomial

$$(x - (2 + 3i))(x - (2 - 3i))?$$

# 2013 Math Bowl

III.10

How many zeros are at the end of  $300!$  ?

# 2013 Math Bowl

## Varsity Math Bowl

### Round 4

# 2013 Math Bowl

Sample:

If  $x$  is an angle such that  $\tan(x) = 3/4$ ,  
what is  $\cot(x)$ ?



# 2013 Math Bowl

IV.1

Simplify – write as a fraction in lowest terms

$$\frac{1! 4! 5! 8! 9! 12! 13!}{2! 3! 6! 7! 10! 11! 14!}$$

# 2013 Math Bowl

Suppose

$$f \circ g(x) = x^2 + 1$$

and

$$f(x) = x - 4$$

$$g(6)$$

Find

# 2013 Math Bowl

IV.3

Two adjacent sides of a triangle have lengths 4 and 6 and the enclosed angle is 30 degrees. What is the area of the triangle?

# 2013 Math Bowl

## IV.4

Consider all the lines that go through at least two of the points  $(0,0)$ ,  $(1,0)$ ,  $(2,0)$ ,  $(0,1)$ ,  $(1,1)$ ,  $(2,1)$ . How many different slopes do these lines have (counting all the vertical lines as having one slope)?

# 2013 Math Bowl

## IV.5

What is the upper right entry of the inverse of the matrix

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

# 2013 Math Bowl

IV.6

Carry out the following multiplication in  
base 5

$$34 \times 43$$

# 2013 Math Bowl

## IV.7

In a triangle with one angle equal to  $22.5$  degrees, a line is drawn through the centroid parallel to the side opposite this angle. This line divides the triangle into a smaller triangle and a trapezoid. What is the ratio of the area smaller triangle to the area of the trapezoid?

# 2013 Math Bowl

IV.8

Simplify:

$$\sin\left(\tan^{-1}\left(\cos\left(\cot^{-1}(2)\right)\right)\right)$$



# 2013 Math Bowl

IV.9

Find the positive value of  $x$  that satisfies:

$$\frac{\frac{x}{2} + \frac{3}{x}}{\frac{x}{4} + \frac{5}{x}} = \frac{3}{2}$$

# 2013 Math Bowl

IV.10

Alan and Billy had some money but Charlie had none. Alan gave him half of his. Billy gave Charlie the same amount of money but it was two thirds of his total. After receiving the money, Charlie now has what fraction of all their money?

# 2013 Math Bowl

Thank You

Awards Ceremony to Start  
Soon