

CALIFORNIA STATE UNIVERSITY, BAKERSFIELD
Lee Webb Math Field Day 2013
Individual Medley, Freshman- Sophomore Level

For each of the following questions, blacken the appropriate circle on the answer sheet. Each correct answer is worth four points. **One point is deducted for each incorrect answer.** An unanswered question is given zero points. Note that random guessing may adversely affect your score.

You have 50 minutes to complete the examination. If you finish early, review your answers. When the exam is over, give your answer sheet to the proctor.

All calculators, cell phones, music players, and other electronic devices should be put away in backpacks, purses, pockets, etc. Leaving early or otherwise disrupting other contestants may be cause for disqualification.

1. In the Federal Court of Eastern California, the jury pool is 10,000,000. Of this number 4,000,000 are OTW. In a randomly chosen jury of 12 people, we should expect how many OTW (round to the nearest whole number)?

- A. 2 B. 3 C. 4
D. 5 E. 6

2. Points A, B, C, D, E, F are such that A, B, C are collinear. Likewise, D, E, F are also collinear (on a different line). How many distinct lines are determined by these six points?

- A. 11 B. 9 C. 15
D. 12 E. 6

3. A square is inscribed in a unit square. One corner of the inner square is $1/4$ away from the corner of the outer square. The length of the diagonal of the inner square is $\sqrt{b}/4$. Solve for b .

- A. 2 B. 3 C. 4
D. 5 E. 6

4. A chord of a circle has length 6 and is 1 unit away from and parallel to a diameter of the circle. What is the length of the diameter?

- A. 7 B. $\sqrt{20}$ C. 8
D. $\sqrt{10}$ E. $2\sqrt{10}$

5. What is the value of the 1st degree coefficient of $(x+1)(2x+1)(3x+1)(4x+1)(5x+1)$?

- A. 1 B. 3 C. 10
D. 15 E. 16

6. Let $f(x) = 3x + 1$. Determine $f(f(f(3)))$.

- A. 10 B. 31 C. 94
D. 271 E. 283

7. Suppose $\frac{2x+1}{5x+2} = \frac{2 \cdot 3 - 1}{4 \cdot 6 - 3}$. Then $34x =$

- A. 5 B. 3 C. -8
D. 15 E. -22

8. Suppose $x \circ y = 2x + 3y$. Determine $(3 \circ 4) \circ 5 - 3 \circ (4 \circ 5)$

- A. 30 B. -29 C. 18
D. 0 E. -24

9. A $3 \times 3 \times 3$ cube is painted on 3 mutually adjacent faces. The cube is then cut into 27 equal $1 \times 1 \times 1$ cubes. One of these smaller cubes is chosen randomly. It is rolled, as a die. What is the probability that the top of the rolled cube is painted?

- A. $1/3$ B. $1/6$ C. $2/9$
D. $3/11$ E. $7/18$

10. Seven vampires and eight werewolves met in a forest. During the meeting, a grizzly bear jumped out and randomly mauled three of them. What is the probability that all three mauling victims were vampires?

- A. $1/13$ B. $7/15$ C. $7/150$
D. $1/15$ E. $3/26$

11. A boat leaves one shore of a river that is half a mile wide. The boat heads straight for the opposite bank at 10 mph. The current in the river is 4 mph. Due to this current, the boat will reach the opposite bank how far downstream?

- A. 1/10 B. 1/5 C. 1/4
D. 1/2 E. 1

12. For a given equilateral triangle, let R be the radius of the circumscribed circle and r be the radius of the inscribed circle. Which of the following is closest to R/r?

- A. 1.5 B. 2 C. 2.25
D. 2.5 E. 3

13. All the solutions to $\frac{x}{y} + \frac{y}{x} = 2$ lie on a

- A. circle B. hyperbola C. ellipse
D. parabola E. line

14. Solve for x: $4^{2x+3}/8^x = 16^{x-1}$

- A. 6 B. 7/3 C. 10/3
D. 3 E. 3/7

15. If two standard dice are thrown, what is the probability that their sum will be a multiple of 3?

- A. 1/3 B. 11/36 C. 13/36
D. 1/4 E. 2/9

16. What is the radius of a circle inscribed in a rhombus that is formed by putting two equilateral triangles together, given that the equilateral triangles have side length 12.

- A. 2 B. 3 C. $3\sqrt{3}$
D. $4\sqrt{2}$ E. $4\sqrt{3}$

17. Suppose we are given the system of equations:

$$x + y + z = 10 \quad x - y + z = 4 \quad x + y - z = 8 . \text{ Determine } xyz .$$

- A. 0 B. 30 C. 36
D. 18 E. 9

18. Suppose Albert and Barney played a series of table tennis matches. There were no ties. Albert won twice as many games as Barney. If one of these matches was chosen randomly, what is the probability that the chosen match was one that Albert won?

- A. 2 B. $1/2$ C. $2/3$
D. $3/4$ E. None of these

19. Two brothers inherit a flock of sheep, which they decide to sell. The price per sheep happens to equal the size of the flock. The money comes in a stack of \$10 bills and some change. They split the money as follows. Joey first takes a \$10 bill, then Josh takes one, and they continue alternating. When the bills are gone, Joey says “you may keep all the change.” But Josh says “this still isn't even because you got more bills than I did.” Joey says “OK, I will write you a check that will make it fair.” What is the amount, in dollars, that Joey should make out the check for?

- A. 1 B. 2 C. 3
D. 4 E. 5

20. One end of a rectangular swimming pool is 10 yards wide and along this end the water is 2 feet deep. At the other end, 20 yards away, the water is 10 feet deep. The depth changes steadily. The walls are vertical. At 1 cubic foot per second, which of the following is closest to the number of hours it will take to fill the pool?

- A. 1 B. 2 C. 3
D. 4 E. 5

21. The sum of thirteen consecutive odd numbers is 299. What is the largest of these numbers?

- A. 5 B. 18 C. 23
D. 35 E. 47

22. Which of these numbers can not be written as the difference of two squares?

- A. 2001 B. 2013 C. 2014
D. 2015 E. 2016

23. Albert, Betty, Carol, and Darshan drove across the country in seven days. On days when Albert sat in back, Betty sat in front. On days when Betty sat in back, Darshan also got in back, but Carol did not. On days when Darshan sat in back, Albert or Carol sat in back too. No two days had the same set of people in the back. For how many days of the trip was Darshan in the back?

- A. 1 B. 2 C. 3
D. 4 E. 5

24. How many zeros are at the end of $500!$?

- A. 100 B. 125 C. 150
D. 175 E. 200

25. A regular icosahedron is a solid with twenty congruent triangular faces. How many vertices does it have?

- A. 12 B. 15 C. 18
D. 20 E. 60

