

CALIFORNIA STATE UNIVERSITY, BAKERSFIELD
MATHEMATICS FIELD DAY 2025

Individual Medley, Varsity Level

There are 25 problems. You will have 50 minutes. You will only have to turn in the answer sheet – you may keep the test. Each correct answer is worth four points. Each incorrect answer will receive a one point penalty. If you believe there is an error of some sort in a particular problem, please quietly discuss it with one of the proctors. For each of the questions, fill in the appropriate circle on the answer sheet. When the exam is over, give your answer sheet to the proctor.

Calculators of any kind are not allowed.

GOOD LUCK!

1. How many integers from 1 to 2025 are divisible by 20 or 25, but not both?
(A) 101 (B) 142 (C) 157 (D) 162 (E) 182
2. What is the remainder when 7^{2025} is divided by 100?
(A) 1 (B) 7 (C) 43 (D) 49 (E) 51
3. If $f(x) = x^3 - 6x^2 + 11x - 6$, what is the sum of the reciprocals of the roots of f ?
(A) 1 (B) $\frac{11}{6}$ (C) $-\frac{11}{6}$ (D) 6 (E) -6
4. A farmer wants to build a rectangular fence using exactly 100 meters of fencing material. What is the maximum possible area (in square meters) that can be enclosed?
(A) 400 (B) 500 (C) 625 (D) 1000 (E) None of these

5. Which of the following are equivalent to $\sqrt{5 + \sqrt{24}}$?
- (A) $\sqrt{2} + \sqrt{3}$ (B) $\sqrt{8} + \sqrt{5}$ (C) $1 + \sqrt{5}$ (D) $\sqrt{10}$ (E) None of these
6. The parabola $f(x) = 2x^2 + ax + 7$ for some unknown a has a vertex at $(2, -1)$. What is $f(-1)$?
- (A) 1 (B) 8 (C) 13 (D) 17 (E) 23
7. Find the product of the zeros of $p(x) = 2x^5 - 4x^4 + x^3 - 2x^2 + 15x - 7$.
- (A) 2 (B) $\frac{7}{2}$ (C) $\frac{15}{4}$ (D) $-\sqrt{7}$ (E) -2
8. A sphere is inscribed in a cube. This cube is then inscribed in another sphere. What is the ratio between the volume of the larger sphere to the smaller sphere?
- (A) $\frac{\pi}{8}$ (B) $\frac{\sqrt{6}}{2}$ (C) 2 (D) $2\sqrt{2}$ (E) $3\sqrt{3}$
9. The Fibonacci Sequence starts $1, 1, 2, 3, 5, 8, 13, \dots$, where starting from the third term, each term in the sequence is the sum of the previous two terms. How many of the first 2025 Fibonacci numbers are even?
- (A) 365 (B) 405 (C) 555 (D) 625 (E) None of these
10. The real valued function $f(n) = \log_2(5n - n^2 - 6)$ is defined for how many integer values of n ?
- (A) 0 (B) 1 (C) 2 (D) 3 (E) ∞
11. A large equilateral triangle is divided into 16 smaller equilateral triangles of equal size. If the perimeter of the large triangle is 48, what is the perimeter of one of the smaller triangles?
- (A) 3 (B) 6 (C) 8 (D) 9 (E) 12

12. Two concentric circles have radii 5 and 13. A chord of the larger circle is tangent to the smaller circle. What is the length of the chord?

(A) 12 (B) 15 (C) 18 (D) 24 (E) None of these

13. How many 3 digit numbers have the property that their digits are written in strictly increasing order?

(A) 80 (B) 81 (C) 82 (D) 83 (E) 84

14. How many distinct real solutions does $x^4 - 8x^3 + 24x^2 - 32x + 16 = 0$ have?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

15. Let a sequence a_n be defined by

$$\begin{aligned}a_1 &= 2 \\a_{2n} &= 2a_n \\a_{2n+1} &= 2a_n - 4a_{n+1}\end{aligned}$$

What is the value of $a_1 + a_2 + \cdots + a_{63}$?

(A) -54 (B) -128 (C) -246 (D) -1216 (E) -2025

16. Simplify $\cos(36^\circ)\cos(72^\circ)$.

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{\sqrt{2}}{4}$ (E) None of these

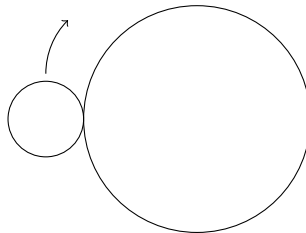
17. How many ways can you arrange the letters in the word "ALGEBRA" so that no two vowels are adjacent?

(A) 144 (B) 576 (C) 720 (D) 1440 (E) None of these

18. A standard 6-sided die is rolled 6 times. What is the probability that exactly one number appears exactly twice?

(A) $\frac{25}{324}$ (B) $\frac{5}{54}$ (C) $\frac{5}{1944}$ (D) $\frac{25}{108}$ (E) None of these

19. A small circle of radius 2 is rotating around a larger circle of radius 6, as in the figure. The larger circle remains static. At the end of how many revolutions will the smaller circle return to its starting point?



- (A) 1.5 (B) 3 (C) 3.5 (D) 4 (E) 6

20. Suppose that x, y , and z are real numbers satisfying

$$x + y + z = 6, \quad x^2 + y^2 + z^2 = 14.$$

What is the value of $xy + yz + zx$?

- (A) 2 (B) 5 (C) 8 (D) 11 (E) None of these

21. A polynomial p satisfies

$$p(p(p(x))) = 8x + 49$$

for all real x . What is $p(3)$?

- (A) 9 (B) 11 (C) 13 (D) 15 (E) 17

22. Suppose $f(1) = 1$ and $f(n) = f(n-1) + \frac{1}{n}$ for any positive integer n . For how many n is $f(n)$ an integer?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) ∞

23. Let S be the set of integers $\{1, 2, 3, \dots, 100\}$. A subset T of S is chosen at random, with each number having an equal chance of being included or not being included. What is the probability that T contains exactly one multiple of 10?

(A) $\frac{99}{512}$ (B) $\frac{25}{256}$ (C) $\frac{10}{99}$ (D) $\frac{10}{11}$ (E) $\frac{5}{512}$

24. Suppose that x and y satisfy the equation

$$x^2 + y^2 = 10x + 6y + 20.$$

What is the minimum possible value of $x + y$?

(A) -8 (B) $3 + 2\sqrt{2}$ (C) $6 - 4\sqrt{3}$ (D) $8 - 6\sqrt{3}$ (E) None of these

25. For how many positive integers n is $n^2 + 7n - 1$ a perfect square?

(A) 0 (B) 1 (C) 3 (D) ∞ (E) None of these